Quantum Reality

By

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Introduction

Why another document on Quantum Reality?

Many years ago when I was at university I alternated between the Mathematics and Physics departments for various subjects. The Physicists were an impressive lot, demonstrating mastery of Electromagnetism, planetary orbits, heat transfer and the like. Their mission was to come up with solutions and mathematics was a tool. Terms in equations were ruthlessly crossed out, substituted or ignored and the final results closely matched observed reality.

But then I would cross over to the Mathematics department where I would be assailed with pathological examples of functions and sets that defied common sense. In the end the mathematicians spooked me enough not to completely trust anything without examining it closely.

I decided to investigate Quantum Reality.

After trawling the Internet and book stores I discovered that there is now a considerable industry in alternative “Quantum Interpretations”, generated by purveyors of healing crystals to Science journalists and academics. Some clearly have a vested interest in the process. Some do not.

The majority of the original founders of Quantum Mechanics would have been bemused; apart from a few holdouts, they had regarded the matter as settled.

With the passage of time it has become more and more difficult to get agreement on what the Copenhagen Interpretation is, if that was ever possible in the first place (Bohr, Heisenberg, Born, Wigner and Von Neumann didn’t always agree on everything). The confusion that increasingly surrounds interpretations of Quantum Mechanics is not helped by modern proponents who seem to be able to find something in the historic writings to back just about any position. Arguing over who-said-what hardly advances the Physics; the author believes that the nuances of the individual views of Bohr and Heisenberg now are mainly of history interest anyway since the Copenhagen Interpretation, or Orthodox Interpretation, has evolved since its original formulation. The version of the Copenhagen Interpretation presented here (London Interpretation ☺) would hopefully have been recognised by Bohr and Heisenberg, but it is undoubtly a modernised variant but few references are provided to justify any “updates”.

This investigation began as a survey (almost everything in the first part of this document can easily be found elsewhere) but in the end, it was almost impossible not to be persuaded by one particular viewpoint. I will leave it to the reader to discover which, and more importantly, why.

This document is divided into four parts:

- Part A deals with various interpretations of Quantum Mechanics currently in circulation.

- Part B deals with the measurement process, which turns out to be a surprisingly rich topic. Part B takes the conclusions of part A as read. I.e. it will assume that one particular interpretation is the correct one and measurement will then be analysed primarily from this point of view.

- Part C presents the London (Ticker Tape) Interpretation of Quantum Mechanics. It is the results of the author's investigations in Part A and Part B.

- The Appendices contain material that can be included or left out dependent on the existing knowledge of the reader. For example, the Quantum Mechanics primer is aimed at readers who do not have a strong Physics background, and the section on Interpretation is the results of numerous emails received over the years. The author's experience is that the failure to grasp the concepts in this appendix is responsible for a great deal of confusion and lack of rigor surrounding Quantum interpretations.

I am not a philosopher or a historian, and this has influenced the work. Mathematics and Philosophical terminology has been kept to a minimum, hopefully without compromising the technical nature of the material.

If you disagree with any statement in this document, please feel free to email me at sokane@bowmain.com

Part A

Interpretations

of

Quantum Mechanics

1 The Contenders

"For those who are not shocked when they first come across quantum theory cannot possibly have understood it." – Niels Bohr[9].

There are as many interpretations of Quantum Mechanics as there are lawyers in the world (too many) so I have chosen only to investigate those that are most popular and which high-light specific features common to all interpretations.

1.1 Copenhagen Interpretation

The Copenhagen Interpretation was developed primarily by Max Born, Niels Bohr and Werner Heisenberg in the earlier years of the 20th Century. It is the dominant view within the Physics community; however it is very difficult to actually find a definitive statement of exactly what the Copenhagen Interpretation is. Bohr and Heisenberg did not agree on all the details so it is not surprisingly to discover that there are multiple subtle versions of it.

Synopsis: The Copenhagen Interpretation is a Positivist interpretation (See Philosophy section). The quantum world can only be examined by constructing machines that perform measurements. Quantum Mechanics is a probability calculus which can be applied to a specific combination of measuring devices and quantum system. The waveform is not "real"; it is a mathematical construction that represents the observer's knowledge of the system. The description of the world is inherently probabilistic; at some point no deeper level of understanding is possible.

1.1.1 What is a measuring device?

This is a really tough question. We shall go with the following definition, inspired by Bohr: A measuring device measures "essentially classical" quantities like position, momentum, energy, charge which "map" to real numbers. Measurements never return complex numbers. To avoid confusion with other uses of the word "real", we shall use ***R*** to indicate the set of real numbers and the term ***R***-valued to indicate a quantity which takes values which are real numbers.

How can we tell what we are measuring? Bohr's view was that the macroscopic structure of a measuring device determines what it measures. There are problems with this view, but whatever its shortcomings, this is the standard view repeated ad nauseum in text books. We shall review this topic later.

1.1.2 Born's Probability Interpretation

What interpretation can be attached to the wave function? The square of the amplitude of the waveform yields the probability density function. This is consistent with experiment (e.g. Diffraction experiments where single photons can be detected) and is not disputed by any interpretation.

1.1.3 Heisenberg's Uncertainty Principle (HUP)

The Copenhagen Interpretation objects to statements such as “It is not possible to measure the momentum of an electron without disturbing its position”*. HUP makes no statement about “disturbing” anything, only that it is not possible to simultaneously measure “conjugate” quantities with arbitrary accuracy. In fact, as we shall see, the position of the electron may not even be well defined.

Heisenberg’s Uncertainty Principle is a consequence of the mathematical structure of Quantum Mechanics and no longer directly challenged by any interpretation.

* Historically this was the view of Copenhagen Interpretation until about 1935 (EPR).

1.1.4 Observer Created Reality

The Copenhagen Interpretation does NOT say that the observer creates reality, whatever that means. It does not say that the world is a purely mental construct (Idealism).

Copenhagen does say that a property of a system however may not be defined until it is measured. For example, if an electrons waveform is smeared out over space, its position is not just unknown but undefined. The electrons position is only defined once it is measured.

1.1.5 Bohr’s Complementary Principle or Wave-particle Duality

Historically quantum particles were said to exhibit a wave-particle duality. This principle says that it is possible to design an experiment to show the particle nature of matter, or show the wave nature of matter, even though each picture of matter is mutually exclusive.

For example, Double Slit Diffraction using a single photon at a time demonstrates the wave-nature of light as the light moves from the source through the diffraction grating and builds up a diffraction pattern on the final screen. The particle nature of light is demonstrated by arrival of discrete packets of energy (photons) at the final screen.

Bohr’s Complementary Principle is at the heart of the Copenhagen Interpretation. Matter is neither a wave nor a particle; each picture is *complementary* not contradictory. It is the nature of the experiment which “chooses” which picture (or aspect) of matter is demonstrated.

The Principle is much more subtle and deep than it first appears; it is best understood by reference to Kantian or Positivist Philosophy. We know the world through our senses, or experimental devices. What we know about the world is as much a product of the devices we use to interact with the “real world” as the “real” world itself, which ultimately lies beyond our senses and essentially unknowable.

The author recommends that the reader re-visit this topic after reading the sections on Philosophy in the rest of this document.

1.1.6 Is the Waveform real?

The standard version of Copenhagen is clear on this issue. Consider the following statement by Bohr.

"*There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.*"

The Copenhagen Interpretation regards the waveform as not real. The alternative interpretation which accepts the same probabilistic view as the Copenhagen Interpretation but regards the waveform as real will be referred to as the State Vector Interpretation.

1.1.7 Waveform Collapse

The Copenhagen regards the waveform as having no reality and is only an aid to calculating probabilities. Consider the following experiment:

A coin is to be thrown from the dark onto a table top. The observer cannot see the coin so the best description he can produce is

$$\Psi = (|heads\rangle + |tails\rangle)/\sqrt{2}$$

The coin tumbles onto the table and settles heads up. The best description of the coin is now

$$\Psi = |heads\rangle$$

At no time did the description affect the outcome of the experiment and so in this sense it has no reality and the "collapse" of the waveform Ψ holds no mystery.

The coin example is also misleading. We are quite happy to accept the "non-reality" of the waveform because we "know" the coin's position and momentum are in well defined states that could be measured, and if the appropriate calculations performed, we could determine in advance which way up the coin would land. The Copenhagen Interpretation on the other hand says the waveform is all that it possible to ever know about the experiment.

1.1.8 Comments on Copenhagen

Many of the original statements of the Copenhagen Interpretation now appear dated and vague.

1.2 State Vector Interpretation

Synopsis: *This interpretation is implicitly presented in many physics text books. It accepts the Born Probability Interpretation and the belief that the ultimate description of the universe is probabilistic. The main difference from Copenhagen is that quantum systems are viewed as being in a state described by a wave function that lives longer than any specific experiment. The waveform is considered to be an element of reality. Measurement implies the "collapse" of the wave function. Once a measurement is made, the wave function is no longer smeared over many possible values; instead the range of possibilities collapses to the known value.*

This is really a family of interpretations. The collapse of the waveform is problematic and not covered by the equations of Quantum Mechanics. In its pure form, the nature of the collapse is left unresolved. To create a new interpretation, "roll your own" collapse criteria. Many variants abound.

1.3 Consciousness Causes Collapse

Synopsis: *It accepts most of the State Vector Interpretation except measuring devices are also regarded as quantum systems. This has some interesting consequences: If the measuring device is a quantum system, the measuring device changes state when a measurement is made but it's waveform does not collapse. So when does the waveform collapse? (See "The Measurement Problem" below) The collapse of the wave function can be traced back to its interaction with a conscience observer. Seriously scary!*

Conscious Causes Collapse is a variant of State Vector Interpretation.

1.4 Many-worlds Interpretation

Synopsis: *All possible outcomes are regarded as "really happening" and somehow we "select" only one of those realities (universes). The Many-Worlds Interpretation is real SF stuff. It is also untestable unless someone comes up with a way of moving from one universe to another. The interpretation was first formulated by Everett.*

If |**a**> and |**b**> are possible outcomes of an experiment, then so is c|**a**> + d|**b**> for any complex numbers c,d where $|c|^2 + |d|^2 = 1$. The total number of alternate universes is therefore infinite. Since "every possible outcome" is viewed as happening, the many world interpretation implies universes exist where the laws of physics hold but every toss of a coin has always resulted in a "tail"! Universes "interfere" with each other "smearing" our universe at the quantum level. Making a measurement "selects" one of these universes.

Proponents of this theory suggest that the many-worlds interpretation of Quantum Mechanics is now the dominant view within the physics community. If true, this is undoubtly due to similarities between many-worlds and Feynman's approach (See below).

1.5 Hidden Variable Theories

Synopsis: *Proponents have philosophically objections to what Einstein would call "God playing dice". It views electrons and other quantum objects as having properties with well-defined values that exist separately from any measuring device (e.g. position and momentum) Quantum Mechanics is viewed as a high level statistical description of the underlying theory. This view was favoured by Einstein and others.*

The terminology "Hidden Variable Theories" is misleading. Many Quantum theories have "hidden" variables within their formulation. A better term would be "Deterministic theories". However we shall maintain what has now become a universally accepted convention.

Hidden Variable Theories may or may not consider the waveform to be an element of reality. I.e. it may be purely a statistical creation, or it may have some physical role (e.g. Bohm).

1.6 Extended Probability and Quantum Logic

Synopsis: *These interpretations attempt to reformulate Quantum Mechanics as probability or logic theories.*

The study of alternative theories of probability would be a valid and interesting area of research even in the unlikely event it was shown to have no relevance to Quantum Mechanics.

Are there alternatives to the standard Kolmogorov probability axioms? One of the more compelling derivations of probability axioms from desiderata is due to Cox[10]. It appears from this derivation that viable alternatives to standard probability theory are limited; however the requirement for probability values to be real numbers can be relaxed. The resulting non-real probabilities correspond well to quantum waveforms, however it is hard to understand why Nature should prefer non-real probability over standard probability.

1.7 Consistent Histories

Synopsis: *This approach was inspired by Bohr's Complementary Principle. The observer chooses a set of "consistent" measurement operators to analyse a specific situation. The choice of operators, called a framework, establishes the set of outcomes that can be talked about. I.e. it establishes a sample space. It can be shown events in sample space can be analysed using standard Boolean logic and probability theory. Events in differently defined sample spaces cannot generally be combined using standard logic.*

Some author's claim Consistent Histories is "Copenhagen done right" since it contains precise statement of principles such as the Bohr's Complementary Principle. However Consistent Histories has many variants and is often expanded to include Decoherence and a "real" waveform collapse. For this reason, Consistent Histories will not be considered separately; the various components of the interpretation will be covered by the analysis of other interpretations. In particular, the Consistent Histories concept of "frameworks" is consistent with Copenhagen and will be subsumed into Copenhagen (with acknowledgement).

1.8 Decoherence

Synopsis: *The waveform is regarded as an element of reality. The measuring device is regarded as a Quantum system. When a measurement is made using "realistic" devices, the interactions of the combined system (target system and measuring device) with the external environment results in the waveform moving rapidly and continuously toward stable 'pure' (eigen)states.*

Caveat: The term decoherence is somewhat ambiguous. It is commonly used in the Many-Worlds interpretation to indicate the selection of a universe (which corresponds to the separation of a superimposed state into the "pure" (eigen)states associated with each universe). That is a discontinuous process.

Decoherence is at odds with the discontinuous behaviour observed in double slit diffraction, the photo-electric effect and many other processes. (For example, in the Photo-electric effect, a metal surface is bathed in radiation. The waveform describing emitted photons is evenly distributed across the entire metal surface, until a photon is emitted discontinuously)

Decoherence has become an increasingly popular concept in recent years, along with the related topic of Quantum Computing. However Decoherence is little more than an engineering specification elevated to the status of an interpretation: a macroscopic measuring device must be engineered so that it (a) moves rapidly to a stable state once a measurement is made and (b) does so by interacting with the external environment and measured system. Not all Quantum systems behave that way. Decoherence suffers from the same problems as all other real waveforms and will not be considered separately.

2 Philosophers, Physicists and a Mathematician

Disclaimer: The philosophical material below is greatly simplified and so may lack many of the subtleties of the original works. I am not a trained philosopher. If you find this area of interest or useful, you should conduct further study elsewhere.

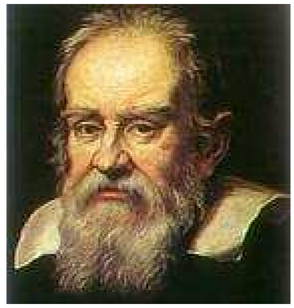
An understanding of the work of philosophers is necessary to understanding much of the published discussion on quantum interpretations. It is also useful in countering those who overstate positions. e.g. “Concepts which cannot be measured have no place in science” or “Science reveals the secrets of nature”. It is also very interesting to discover just how long it took for an accurate view of what Science actually is to develop.

To appreciate the contribution of various philosophers it is necessary to understand the historical context. At the beginning of the renaissance, the pursuit of scientific knowledge was not distinct from the pursuit of theological knowledge. Since nature is the work of God, every scientific idea has theological consequences and was subject to the scrutiny of the Church. It is fair to say that the Church adopted positions based on highly subjective interpretations of scripture. One of the most famous examples of this was the rejection of the Copernicus' view that the sun was at the centre of the universe, and not the earth. The Bible was regarded as clearly stating that man was at the centre of the universe and so Copernicus simply had to be wrong. (E.g. Joshua 10:12-14. God stopped the sun in the heavens and extended the daylight hours for Joshua and his army to finish the battle with the Amorites). The views of the Church could not be taken lightly. It suppressed the idea until (and some time after) the invention of the telescope showed moons orbiting Jupiter. Clearly not everything revolved around the earth after all.

The following philosopher/scientists made notable contributions.

2.1 Francis Bacon (1561 - 1626)

Bacon denounced the reliance on authority and argument divorced from the real world. He called for reasoning based on inductive generalization of observation, experiment and hypothesis. Bacon is famous for dying from Pneumonia, contracted while stuffing a chicken with snow to see if it could be used as a preservative.

2.2 Galileo (1564 - 1642) 

Galileo was one of the great scientists of the Renaissance, and is sometimes called the “Father of Physics”. His battle with the Church over the Copernicus’ view of the universe is described in the opening paragraphs of this chapter. The books he wrote spread new thinking across Europe and helped to loosen the Church’s grip over scientific matters. Amongst his many achievements is an early statement of the Principle of Relativity.

2.3 Rene Descartes (1596 - 1650) 

“I think (drink) therefore I am, but I don't know about you guys”

Everyone who has been to university bar with a group of drunken friends has heard this one before. “How do I know you are real? You could all be part of my imagination. I might wake up from a dream and you will disappear”.

Descrates also rejected the role of authority, but choose mathematics and deductive reasoning as the basis for his approach to science. He refused to accept any belief unless he could “prove” it. He used his famous “I think therefore I am” to “prove” his own existence, and then went on to “prove” the existence of God. For those who are interested, one of Rene's proof goes something like: I can imagine perfection, and God is perfect. To not exist would be an imperfection, therefore God exists. Hmmm.

2.4 Immanuel Kant (1724 - 1804) 

The following is part of a translation of Immanuel Kant's "The Critique of Pure Reason" [6]

"*all our intuition is nothing but the representation of phenomena; that the things which we intuite, are not in themselves the same as our representations of them in intuition, nor are their relations in themselves so constituted as they appear to us; and that if we take away the subject, or even only the subjective constitution of our senses in general, then not only the nature and relations of objects in space and time, but even space and time themselves disappear; and that these, as phenomena, cannot exist in themselves, but only in us. What may be the nature of objects considered as things in themselves and without reference to the receptivity of our sensibility is quite unknown to us*"

This could have been written by Niels Bohr 150 years later. Kant was, of course, not attempting to understand the meaning of a new and strange theory; he was discussing the limits of human knowledge. In Kant's day it was not uncommon for philosophers to make meta-physical assertions and arguments like Descrates' "Proof of the existence of God". In modern language, Kant pointed out the difference between the internal mental models we build of the external world, and the real objects themselves which we know only through our senses. We can apply logic to our mental models, "*whilst the transcendental object remains for us utterly unknown*".

It is a measure of the success of Kant's views that arguments such as Descrates' would today be met with howls of derision. The division between Science and meta-physics is now clearly established; Science relates to that which we can intuit through our senses, meta-physics and religion relates to that which we cannot.

2.5 Positivism (early 20th Century)

Positivists take Kant's position even further; they reject meta-physical statements as utterly meaningless. Religion, for example, is excluded from philosophy. Positivists regard a statement as meaningful only if it can be verified. Statements can be verified if they are experimental propositions, or if experimental outcomes can be deduced from them. Experimental propositions are judged as verified if they are probable based on experimental evidence.

Positivism clearly influenced the Copenhagen interpretation: Only those "properties" of a quantum system that can be measured, or manifest themselves in measurable quantities, are meaningful. (Measuring devices take the place of Kant's "senses").

2.6 Niels Bohr (1885 - 1962) 

Bohr was both a philosopher and a scientist. Bohr is forever linked with the Copenhagen Interpretation which he was largely responsible for. Bohr produced one of the first quantum models of the atom, engaged Heisenberg and others in debate over the nature of Heisenberg's work, and defended the Copenhagen Interpretation against attacks by Einstein.

2.7 Sir Karl Popper (1902 - 1993)

Scientific enquiry was regarded as primarily inductive in nature up until this point. Popper's view was that scientific theories are axiomatic structures which are subject to falsification. I.e. they cannot be proved correct, but they can be proved incorrect. Science is viewed as a producing a series of theories which are better and better approximations to the "truth".

2.8 Kurt Gödel (1906 – 1978) 

Kurt Gödel was a mathematician, not a physicist or philosopher. Gödel's celebrated Incompleteness Theorems says (greatly paraphrased):

1. Any non-trivial theory contains statements which cannot be proved correct or incorrect from the axioms of the theory.
2. No theory can prove its own consistency (lack of contradiction).

This would *seem* to imply that it is not possible to construct a "Theory of Everything". The best we could do is a "Theory of Almost Everything", which we could never prove wasn't self-contradictory! Even if that was only because we couldn't prove the mathematics we are using isn't self-contradictory!

Kurt Gödel great contribution to reduce the expectations of Mathematicians and the practioners of all disciplines based on Mathematics.

2.9 Thomas Kuhn (1922 - 1996)

Kuhn placed Science in a social context; he was sceptical of the Popper's idealistic view of how scientists work. Scientists are mostly problem solvers, working within the scope of an all encompassing and highly successful theoretical structure, or paradigm, such as Newtonian Physics, or General Relativity, or the Central Dogma of Molecular Biology. Few venture outside the framework, spending most of their time applying the rules of the paradigm to the problems they have to solve. Only occasionally do “revolutions” such as Relativity or Newtonian Physics come along and change the paradigm. The idea that Science “reveals” the truth is rejected.

2.10 George Polya (1887 - 1985)

Polya placed “plausible reasoning” (i.e. reasoning using probability) at the heart of not just scientific thinking, but all of human reasoning. Polya has analysed Scientific and Mathematical advances to show the operation of probability syllogisms.

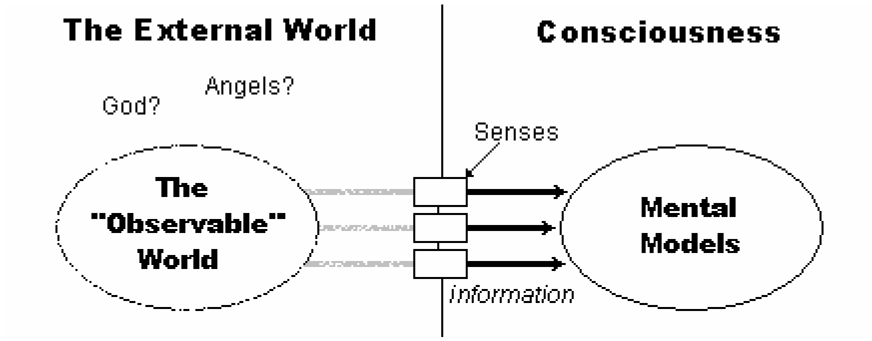
Bayesian probability provides a framework for understanding previous ad-hoc attempts at codifying scientific endeavours. Scientific theories are indeed axiomatic structures capable of falsification, but individuals make judgments on the correctness of theories based existing knowledge and beliefs. Experimental evidence may simply fail to convince sceptics; there is almost always some experimental evidence which contradicts even the most successful of theories. The new evidence may prove to be false, or an existing theory modified to accommodate the new information. Bayesian analysis shows that the die-hard adherents to older theories are behaving rationally given their existing a-prior beliefs.

"A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.” - Max Plank (Autobiography)

“*It is more important to have beauty in one's equations than to have them fit experiment ... because the discrepancy may be due to minor features that are not properly taken into account and that will get cleared up with further development of the theory ... It seems that if one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress*” - P.A.M. Dirac.

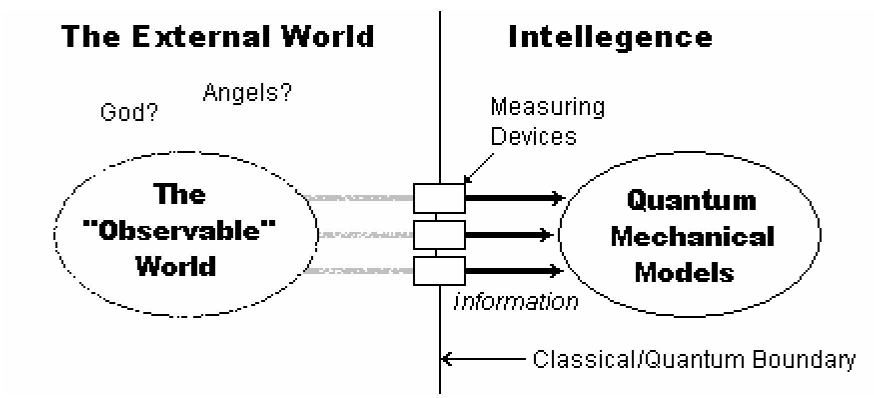
2.11 Kant and Copenhagen

Copenhagen is one of two Quantum interpretations of Mechanics firmly routed in classical Western philosophy. The Kantian world-view is show in the diagram below. Information makes its way from the external world into our consciousness via our senses. We use that experience to construct mental models that predict the behaviour of the external world. If the models are good ones, it becomes easy for us to forget that we are reasoning only with mental constructs and the true nature of the external world is hidden from us.



Kantian World-View
Figure 2.11 (a)

The Copenhagen world-view is shown in the diagram below. The similarity is striking. Information flows from the external world through "measuring devices". Quantum Mechanics goes further than Kant: it tells us which characteristics of the measuring device are important, how measuring devices interact and how to reason with the results. It tells us that Kant's world view does not require consciousness, only "intelligence" - a computer will do ever bit as well as a conscious being. It tells use we need to select a set of measuring devices that the intelligence uses to interact with the world (called unfortunately the "Classical-Quantum Boundary")



Copenhagen World-View
Figure 2.11 (b)

2.12 Realism and Hidden-Variable Theories

Hidden-variable theories are closely aligned with the Philosophical tradition of Realism – the belief that the world is pretty much how it appears to be. The universe is viewed as a vast machine made from "wheels and cogs" (fields and particles) with "everyday properties" (position, momentum, etc); the job of Physics to accurately describe those wheels and cogs and the way they interact.

3 Paradoxes, Phenomena and Proofs

3.1 The Waveform that refuses to collapse - The Measurement Problem

The Measurement Problem: Suppose, for consistency, we decide to treat the measuring device as a quantum device, then the waveform associated with any experiment never collapses!

This is not too hard to see. Suppose we have a radioactive source which has a 50-50 chance of emitting a particle within a given time frame. We set up a quantum measuring device to detect the possible emission of a particle. At the end of the specified time, the waveform describing the *measuring device* is in a state which is a 50-50 mix of "particle detected" and "particle not detected"! I.e.

$$(|\text{detected@1}\rangle + |\text{not detected@1}\rangle)/\sqrt{2}$$

We can attempt to set up a second measuring device to extract the measurement from the first measuring device. It too is a quantum system so after the experiment we find it is still in a 50-50 state:

$$(|\text{detected@1}\rangle|\text{device 2 shows detected@1}\rangle + |\text{not detected@1}\rangle|\text{device 2 shows not detected@1}\rangle)/\sqrt{2}$$

In fact it is possible to add an infinite number of cascaded measuring devices but the waveform stubbornly refuses to collapse. I.e. it is always in a mixed state.

This is what the various interpretations have to say.

3.1.1 Copenhagen Interpretation

What is this waveform collapsing rubbish? Measurements are a *real* physical process, however the waveform is only a mathematical construction used to predict the probability of outcomes for specific experiments. It does not "collapse" since it is not *real* and does not have any meaning outside the context of a specific experiment. Asking "What makes it collapse?" implies both *reality* (whatever that means) and a lifetime greater than the original experiment.

It is for this reason that the Copenhagen Interpretation draws the distinction between (a) the quantum system under observation and (b) the external observer. Measuring devices exist on the *Quantum-Classical* boundary. The quantum system is described in terms of probabilities until the measurement is made (a *real* physical process), the results of the measurements can then be processed classically using Boolean logic.

3.1.2 State Vector Interpretation

Don't know, it just does, umm, when it interacts with the measuring apparatus.

In its purist form, State Vector Interpretation does not specify when the waveform collapses. I.e. the interpretation is incomplete. However there are many variants where an arbitrary collapse criterion is postulated.

3.1.3 Consciousness Causes Collapse

For consistency, all devices should be treated as quantum devices; however we know when we "look" at the outcome of an experiment we always see an outcome, i.e. the waveform has collapsed.

Suppose we consider a simple electron position measurement experiment. We know the waveform does not collapse when the quantum measuring device initially "measures" the electron's position. We know the human eye can also be considered a quantum system, so the waveform does not collapse when photons from the measuring device interact with the eye. The resulting chemical signals in the brain also can be viewed as a quantum system so it is not responsible for the collapse of the waveform. However a conscious observer always sees a particular outcome. I.e. the waveform collapse can be traced back to it's interaction with the consciousness of the observer!

Perhaps a human observer causes the collapse of wave functions, but what about a parrot, or a bacterium? Did the Universe behave differently before life evolved?

3.1.4 Hidden Variable Theories

Measurement does just that. It reveals the values of underlying quantities (usually, although this depends on the theory). Until the more sophisticated theory underlying Quantum Mechanics is discovered it is not possible to make any definitive statements. Individual theories can be tested against experiment and the current formulation of Quantum Mechanics but failure of a specific theory does invalidate the quest for such a theory.

3.1.5 Many Worlds Interpretation

All the different possibilities happen, each in a separate "parallel" universe. I.e. there are at least 2 parallel universe, one with the particle emitted and detected and one were it has not.

3.1.6 Consistent Histories (Extended Copenhagen)

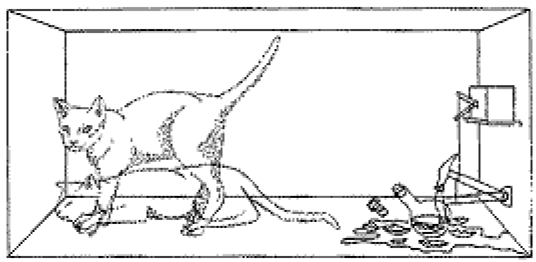
A sample space (framework) needs to be defined before any analyse is done. In the case above, a suitable choice would be

S = { |**not emitted**>|**not detected**>, |**not emitted**>|**detected**>, |**emitted**>|**not detected**>, |**emitted**>|**detected**>}

There is no infinite regression of measuring devices. The choice of framework creates a quantum-classical boundary. Once an observer chooses to restrict his information to that provided by the framework, he enters the classical domain. The probability of these events can be manipulated using standard probability theory. I.e. the events can be treated as "real". A quantum measurement becomes indistinguishable from a classical measurement.

3.2 Schrodinger's Cat

Synopsis: Schrodinger published the following paradox in 1935. A cat is placed in a box for a period of time with a radiation source. During that time there is a 50-50 chance of a particle will be emitted by the radiation source and be detected. If it is detected, a poisonous gas will be released killing the cat. Just before the end of the experiment, the quantum mechanical wave function describing the cat is a 50-50 mix of live and dead cat. The cat is in a state which is half alive and half dead!



Schrodinger’s Cat

This paradox highlights the status of the waveform. Is it “real”? This is discussed in more detail in Wigner’s Friend.

3.3 Wigner's Friend

Wigner (1961) asks his friend to watch an experiment for him and leaves. When he returns he asks his friend what the result of the experiment was. If he views his friend as part of the experiment and therefore a quantum system, the total system is in an indeterminate state until he asks his friend for the result of the experiment. However, his friend is certain that he was not in an indeterminate state prior to Wigner’s enquiry!

This is really a version of Schrodinger’s cat with a talking cat.

Note that Wigner’s friend could be replaced by a simple recording device. In this case we would have no trouble imagining the recording device waveform in an indeterminate state without any paradox arising. It is often stated that this paradox highlights that consciousness must be given special treatment in Quantum Mechanics!

How do the different interpretations handle this “paradox”?

3.3.1 Copenhagen Interpretation

The waveform has no reality and so has no effect on any friend or cat. There is no paradox.

3.3.2 State Vector Interpretation

If the friend is viewed as part of the quantum system, he is indeed in an indeterminate state with respect to Wigner (i.e. Wigner does not know how his friend will answer) and waveform collapse will occur when his friend answers.

If his friend is viewed classically (i.e. he is not part of the quantum system), the waveform collapses when his friend observes the experiment.

If the waveform is "real", then it is reasonable to assume that the waveform "really" collapses only once for all observers at the same point in space-time.

The paradox arises because the first part of the argument assumes that the friend is part of the quantum system, and the second part of the argument assumes that his friend is not (i.e. he is a classical observer). The paradox is resolved if all observers agree on common (and correct) conditions for waveform collapse. State Vector Interpretation does not attempt to specify the conditions under which collapse occurs and so State Vector Interpretation is incomplete (and therefore unsatisfactory).

3.3.3 Consciousness Causes Collapse

Consciousness Causes Collapse = State Vector Interpretation + waveform collapse when the waveform encounters consciousness. Since Wigner's friend is a conscious being, the waveform collapses once he (or she) interacts with the experiment. There is no paradox, however Consciousness Causes Collapse suggests that standard Quantum Mechanics (Wigner's friend is in a superimposed state) is wrong.

3.3.4 Many World Interpretation

Many-Worlds interpretation has the same issues as State Vector Interpretation, with "*selection of our Universe*" replacing "*waveform collapse*". Many-Worlds interpretation does not specify the conditions under which selection of our Universe occurs and so it incomplete. Variants abound - e.g. it is possible to combine Many-Worlds interpretation with Consciousness Causes Collapse so that our Universe is selected once a conscience being interacts with an experiment.

3.3.5 Oops!

One wonders what happens if Wigner walks away and falls into a black hole. The waveform would never collapse! If the waveform is "real", would the system remain in an indeterminate state forever?

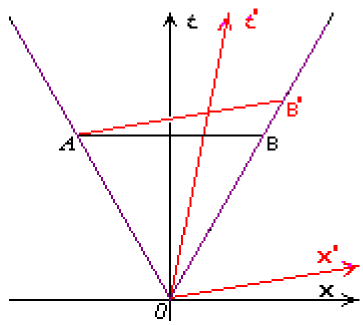
3.4 EPR

Although Einstein was one of the founders of Quantum Mechanics, he was unhappy with many aspects of the theory and famously remarked "*God does not play dice*". Initially Einstein attempted to show that Quantum Mechanics was inconsistent by attempting to circumvent HUP. He was not successful and abandoned that approach in favour of attempting to show Quantum Mechanics is "incomplete". The result was a paper published with the Podolsky and Rosen (EPR) that, in the author's opinion, did a pretty good demolition job on a great many interpretations.

An EPR experiment, shown below, involves any two entangled particles (e.g. photons, electrons etc) emitted from a single source.

EPR Apparatus

The Minkowski diagram below shows the world line of two particles (purple) moving outward from O. The first measurement is made at A. The diagram show 2 frames of reference, a “stationary” frame shown in black, and a “moving” frame shown in red.



EPR Minkowski Diagram

The waveform associated with the emitted particles is in a mixed state (i.e. properties are not well-defined) until the first measurement in made at A. Conservation laws require that the two particles have opposite momenta/spin, so the first measurement at A causes the waveform of the second particle at B to collapse also. Information about the result of the measurement at A propagates instantaneously (faster than light) across space to other entangled particle! EPR argue this shows either Quantum Mechanics is a “non-local” theory (relies on faster-than-light propagation of information), or “incomplete” since it does not describe the mechanism used to maintain momenta/spin conservation.

The argument is sharpened by considering an observer at B' who is moving with a constant velocity with respect to the observer at B. Relativity denies the concept of simultaneity - if the waveform is real, then (presumably) the waveform collapses for everyone at the same points in space-time. However an observer at B believes the waveform collapses before the observer at B' sees the measurement at A! *The instantaneous collapse of the waveform requires us to choose a preferred frame of reference!*

Does collapse occur along the space-like surface AB or the space-like surface AB"? Or along some other space-like surface?

What do the various interpretations say?

3.4.1 Copenhagen Interpretation

The waveform is not “real”, is observer dependent and linked to a specific experiment. The initial waveform associated with observer A initially does not indicate values for momenta/spin for either particle. Once a measurement is made at A, a new waveform is associate with observer A which indicates the new measured value of momenta/spin, AND the values of momenta/spin of the particle at B implied by conservation laws.

Although observer A now knows the momenta/spin of the second particle at B, it cannot communicate this to observer B. Observer B in unaware of observer A's waveform, and his waveform does not indicate the values for momenta/spin measured at A. If he chooses to measure some property then the value obtained will be that demanded by any relevant conservation law, but otherwise random.

Since the separate waveforms do not interfere with each other (they are not "real"), there is no problem.

EPR were correct: Quantum Mechanics is incomplete in the sense that it does not describe the mechanism that "enforces" conservation laws (or the underlying symmetries); however Quantum Mechanics is complete in the sense that no other description is possible.

3.4.2 State Vector Interpretation and other Real Waveform Theories

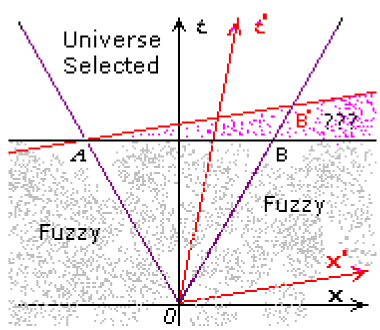
EPR poses serious problems for State Vector Interpretation.

EPR shows State Vector Interpretation to be incomplete (because it is) AND non-local. It is this incompleteness however that prevents further attacks. State Vector Interpretation does not specify collapse criteria.

3.4.3 Many World Interpretation

EPR poses even more serious problems for Many-Worlds interpretation since there is less weasel room.

Many-Worlds interpretation implies that the entire Universe splits into multiple copies of itself when the entangled particles are emitted. The moment the first measurement is made, a particular universe is selected. Does this selection process occur along the space-like surface AB or the space-like surface AB'? Or along some other space-like surface? (Universe selection = waveform collapse) Many-Worlds interpretation does not have a get out-of-jail card - selection of a Universe has or has not occurred at any point in space-time.



EPR/Many-Worlds interpretation Minkowski Diagram

EPR definitely shows that Many-Worlds interpretation is non-local AND incomplete. It requires some definitive mechanism to determine the boundary between the fuzzy pre-selection soup of possibilities and the selected universe.

3.4.4 Can EPR be used as a basis for faster-than-light communication?

No.

Suppose a source that generates particle pairs with random "coupled" spin is placed halfway between the earth and the moon. Even if measuring the spin of a particle arriving on the earth instantaneously causes the waveform "to collapse" on the moon, it is still not possible to detect that collapse since the actual measured values are randomly distributed!

3.4.5 Reality

The EPR paper contained an interesting definition of reality:

"If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity"

After the measurement of a particles position its momentum is not well defined. I.e. its value cannot be determined with probability equal to unity. According to the definition there no longer exists an element of physical reality corresponding to the particle's momentum.

The "unreality" of a particles position and momentum is a consequence of HUP and a particular definition of reality! For any continuous property X of a quantum system, there is always another property $Y = \frac{\partial}{\partial X}$ which is complementary to X and so we are lead to the conclusion that no particle has any permanently ***R***-valued measurable properties!

3.4.6 Comment

EPR is one of the most persuasive arguments in favour of the unreality of the waveform.

What role does "information" plays in Quantum Mechanics? There are 2 particles, but because of conservation laws, their momenta/spin is described by a single "bit" of information. A measurement provides the value of that bit of information. It does not matter which particle is measured first. In fact, it has already been noted that which particle is measured first depends on the observer's frame of reference. Relativistic arguments make it questionable whether that the idea that an influence "travels" superluminally between the particles makes sense.

3.5 Von Neumann's Impossibility Proof

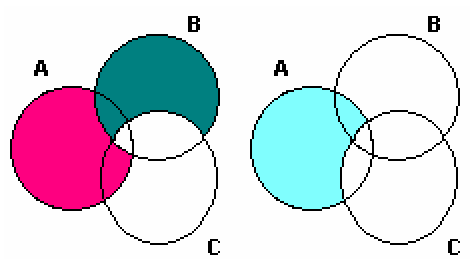
This theorem states that no hidden variable theory is consistent with Quantum Mechanics. At the time of publication it seemed that this proof sealed the fate of hidden variable theories, however this is now disputed by some authors. Criticisms of Von Neumann's Impossibility Proof are generally very technical in nature. Bell's Inequality below is simpler to understand and possibly "does the job better" where the job is to discredit hidden variable theories by imposing very stringent conditions on them.

3.6 Bells Inequality

In 1964, Bell published a paper containing his famous inequality. If a system consists of an ensemble of particles having three Boolean properties (observables) **A**, **B** and **C**, then classically the probabilities associated with selecting particles with various properties obeys Bell's Inequality.

Bell's Inequality: **P(A, ¬B) + P(B, ¬C) ≥ P(A, ¬C)**

Bells Inequality can be seen by examining the Venn diagrams below. The LHS shows the sets A ∩ ¬B (pink) and B ∩ ¬C (green). The RHS shows the set A ∩ ¬C (blue).



Bells Inequality also applies to Hidden Variable theories - the set of states can be modelled by the same Venn diagram; A (set) now represents the set of states that would yield a positive result if a measurement of (observable) **A** were made, *before any measurement of **B** or **C**.*

The problem with applying the inequality to Quantum Mechanics is that measuring one quantity may affect the measurement of another - it is generally not possible to measure the initial values of both A and B for a single particle. However, entangled particles give us two bites of the measurement apple. Suppose we have an EPR style apparatus:

1. There is a reciprocal relationship between the values of measurement of A on the two particles. I.e. there is a conservation law that allows us to predict the value of a measurement of A on one particle given knowledge of the outcome of a measurement on the other.
2. The same type of relationship exists between the particles with respect to quantity B.
3. The value of A is measured for one particle and found to be a.
4. The value of B is measured for the other particle and found to be b.

The first particle **must** have started with state (A=a, B=¬b).

Common sense would indicate that Bells Inequality must be true, however it turns out that Quantum Mechanics does not obey Bells Inequality.

3.6.1 Example

An EPR style apparatus emits entangled photons that then pass through separate polarization analysers (See figure 2 below). Let A, B and C be the events that a single photon will pass through analysers with axis set at 0^o, 22.5^o and 45^o to vertical respectively.

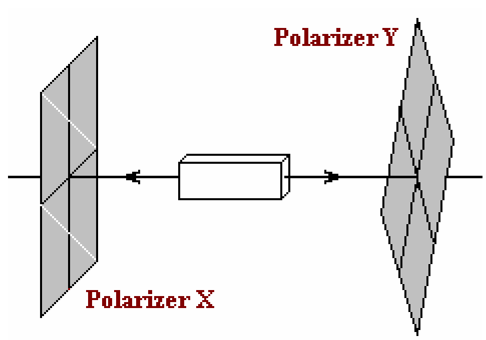


Figure 2.
EPR style polarizer setup.

$P(X, \neg Y)$ = probability that a photon will pass through an analyser **X** but not analyser **Y**. According to Quantum Mechanics, $P(X, \neg Y) = ½ \sin 2$ (angle between analysers);

$$\text{[Bell's Inequality] } P(A, \neg B) + P(B, \neg C) \geq P(A, \neg C)$$

$$\Rightarrow ½ \sin^2(22.5^o) + ½ \sin^2(22.5^o) \geq ½ \sin^2(45^o)$$

$$\Rightarrow 0.1464 \geq 0.25$$

which is obvious nonsense. I.e. Quantum Mechanics is inconsistent with Bell's Inequality.

Experimental evidence supports Quantum Mechanics, not Bells Inequality. *Did anyone seriously believe that Quantum Mechanics, one of the most accurate and successful theories of all time, is incorrect in this simple case?*

3.6.2 Conclusion

Bells Inequality shows it is not possible to simply view particles as having properties attached to them in a classical manner - otherwise Quantum Mechanics would be forced to obey Bell's inequality which it does not. If property A is measured, it is not sufficient to say that the value of B is no longer known; it is no longer possible to say that any value of property B is associated with the particle.

Quantum Mechanics does not support a locally real particle model. Successful hidden variable theories will have to be sufficiently sophisticated to take this into account.

3.7 Closed Systems

The Universe is a closed system. By definition there can be nothing outside it.

Quantum Mechanics has become an integral part of Cosmology; the entire Universe is treated as a quantum system. What does the waveform describing the evolution of the entire Universe look like? Here's what the various interpretations say.

3.7.1 State Vector Interpretation

When does the waveform collapse? State Vector Interpretation is incomplete; different variants have different collapse criteria. Those variants that specify that collapse occurs as a result of external measurements require a measurement external to the entire Universe before collapse can occur! Quantum waveforms exhibit symmetry so postponing collapse has consequences: the Universe would exhibit similar symmetry until collapse occurs for the first time (around Tuesday at 3:20 pm).

3.7.2 Consciousness Causes Collapse

The waveform cannot collapse until it encounters an external consciousness!

3.7.3 Copenhagen Interpretation

The waveform only describes the probabilities. At some point the experimenter must decide what the experimental outcomes are, in which case he creates a quantum-classical boundary. The waveform described the evolution of the Universe prior to the designated outcomes. There are no issues associated with this interpretation or any of its variants

3.8 Consciousness

A number of well respected scientists have conjectured that it may be possible to explain consciousness using concepts from Quantum Mechanics. Typically consciousness is equated with quantum tunnelling across synaptic clefts and the like. The problem with all such attempts to reduce consciousness to particular type of event within the brain is that the same events typically happen outside the brain. For example, if consciousness is the result of quantum tunnelling in the brain then if quantum tunnelling occurs outside the brain, say inside a radar gun, does the radar gun become conscious? *Luke, perhaps the rocks and trees and air some sort of low level consciousness they do indeed have.*

There are alternatives.

It is sometimes claimed that consciousness magically appears once a system's complexity grows beyond a certain point. That's wishful thinking. The idea can be challenged on a number of grounds - it is possible to point to many extremely complex systems which display no indication of consciousness (e.g. telephone networks, a kidney, the Internet), and a complex systems that display rudimentary intelligence (e.g. large software systems) that can be rendered useless by a single flaw. Proponents usually patch up the hypothesis by the addition of arbitrary criteria for distinguishing those systems that *might* be conscious from those that clearly are not (The addition of ad-hoc criteria is very reminiscent of the situation with respect to waveform collapse)

Another possibility arises from work on artificial intelligence. Rodney Brooks suggests that the "*Universe is its own best representation*". Brooks' approach is to build robots that simply process the information from its sensors to do very specific 'simple' jobs. A robot insect might respond to light or noise by behaving in a certain way (e.g. turn away from the light or noise, or lunge at "food"). The result can be surprisingly complex and apparently intelligent behaviour - but the intelligence (consciousness?) is a result of *both* the robots reflexive actions and its external environment.

There is some support for this hypothesis. Animals, driven by "instincts", display complex caring behaviour towards their offspring; however the apparent mechanical nature of this care can be exposed in unusual circumstances. Birds, for example, may refuse to feed hatchlings if they fall out of the nest. The hatchlings may only be inches away from their siblings but the parent bird will simply ignore them. It would be interesting to know how much human behaviour is really of this kind.

A robotic vision system built on Brook's principle might consist of a number of simple systems (edge enhancement, stretch / match current image against image library etc). Many alternative approaches attempt to construct a complete wire-frame style internal model of the universe, and identify features by matching sensory data against the model, but this has proved extremely difficult to do. The attraction of the internal model approach is that once the model has been constructed and matched against the real world, information such as facial features can be easily extracted from it. Using wire-frame style models has proved very difficult to do

Humans however don't seem to need a mental model of a human, or even an animal, to identify facial features. Humans see faces everywhere from cars to the Mars, suggesting that the part of the brain that identifies faces operates directly on (possibly pre-processed) sensory input, as per Brooks, and does not use an internal model. Internal models are also fragile; they attempt to reduce the world to abstract concepts familiar to programmers and mathematicians such as sets, logical propositions etc. An internal model of a person may include "slots" for two legs. It may work wonderfully 99% of the time. It may even handle amputees. But would it handle Jake the Peg? Furthermore, if a system is based on logical inferences, the consequences of a mistake can provoke a massive storm of revisions. Brooks' systems use the fact that constraints are modelled implicitly in the data from the real world - systems that process that data continue to work even if there are radical changes to the environment (but potentially give the "wrong" result). Developmental Psychology favours Brooks**:** human babies are almost totally dependent on sensory input and only gradually relinquish that reliance. (E.g. take 2 fat glasses of Orange Juice filled to the same height. Ask a 4 year old: "which has more?" Now pour one of the glasses of Orange Juice into a tall thin glass. Ask the 4 year old again: "which has more?") It is typically not until late adolescence that humans are fully prepared to use logic to override their senses. (See Jean Piaget)

Brooks approach is fascinating since it implies that intelligence is contextual and cannot be localised as part of a "machine"; it is also a function of the external universe. If Brooks is correct and the "*Universe is its own best representation*", intelligent systems cannot exist without a deep connection with the external world. True artificial intelligence requires not a disembodied logical machines like HAL 9000 in "**2001 - A Space Odyssey"**, but something more reactive with capabilities akin those of the artificial humans in the **"The Stepford Wives"**.

Perception in conscious entities is also raises questions. Physicists know that light is yellow if its frequency lies in a specific range, but such a description is incomplete. The true essence of

Yellow has to be experienced, something someone who is colour blind can never do. Nor can Yellow be experienced by anyone with damaged optic nerves or damage to parts of the brain responsible for processing visual information. Even if we were aware of every single physical and biochemical interaction in the "normal" perception of Yellow, would we be able to pin-point the point were the consciousness of that person perceives Yellow? The ghost of Kant is everywhere; behind the mental models lays a transcendental object, human consciousness, forever beyond linguistic description, which we know exists simply because we experience it ourselves.

Consciousness requires at least intelligent behaviour and perception, but argument, biology and CAT scans suggest that neither of these components is a single thing. (Which part of the brain triggers waveform collapse?) Brookes suggests that it may not reside in any identifiable part of any system. It is possible that consciousness has no explanation. It might just "be". To equate it with any Quantum Mechanical phenomena, or use it as a mysterious agent of waveform collapse, at this stage in the development of human knowledge, seems reckless.

3.9 Free Will

Copenhagen views the world as inherently unpredictable - there is room for Free Will. It is possible that an experimenter "plays dice" with God each time he performs a measurement. But it is equally possible that our fates are pre-ordained like players in a Greek tragedy.

Copenhagen is essentially a Positivist interpretation. Issues of "Free Will" are meta-physical considerations. Copenhagen has nothing to say.

Hidden variable theories however imply a clockwork universe, wound up by God at Creation and unfolding in a predetermined way. There is no true randomness, only a lack of knowledge of initial conditions. All of human history was pre-ordained from the beginning of Time. There is no free will. It follows that hidden variable theories are in conflict with Christian dogma.

3.10 What value Hidden Variable Theories?

A successful hidden variable theory would make the same experimental predictions as standard Quantum Mechanics, while adding extra layers of unverifiable complexity. Even Einstein is reputed to have referred to at least one such theory (Bohm's) as an "unnecessary superstructure". The un-provable nature of hidden variable theories leaves them open to the claim that they are not Scientific.

It is also known that any hidden variable theories must also have "unsatisfactory" features such as non-locality.

Do Hidden variable theories therefore have any real value?

Hidden variable theories will always be preferred by those who in their hearts believe in Realism. They have a naive appeal that simply cannot be overcome by sophisticated argument.

There are even a number of circumstances where a hidden variable theory would add value. Firstly the theory could be so simple and elegant that it becomes compelling, even if it could never be verified. Bell's Inequality and Von Neumann’s Impossibility Proof however make this highly unlikely; they ensure that it would not be a simple particle model. Secondly a successful hidden variable theory might provide a useful calculation methodology; the calculation methodology could come to be regarded as an interpretation in its own right. (Something similar to this has already happened to Feynman’s Sum of Histories) Thirdly a hidden variable theory might be able to explain phenomena that standard Quantum Mechanics could not. For example, a hidden variable theory might be able to predict the value of coupling constants, while standard Quantum Mechanics cannot. In this case, the Copenhagen Interpretation would have to be regarded as wrong.

So far no-one is close to producing a hidden variable theory to rival standard Quantum Mechanics, never mind one that has more power than standard Quantum Mechanics.

4 Probability

4.1 Frequentists vs. Bayesians

Probability theory as a branch of mathematics is several hundred years old. It would be reasonable to expect that probability theory is a stable, well understood branch of mathematics, perhaps with a small number of outstanding complex problems. But this turns out not to be so. There is a schism within Probability theory that matches the schism within the Physics community over Quantum theory. There are two main camps: Frequentist and Bayesians, while the great mass of practioners follow Feynman's dictum (apparently actually Dirac’s dictum) “*Shut up and calculate”*. Both approaches (generally) give the same results, although one approach may be significantly simpler than the other.

Frequentists consider that probability is essentially about “frequencies”; probabilities describe the long term likelihood of an event. Frequentists believe that the goal of probability theory is to correctly estimate the “propensities” (Popper) of a physical system. Probabilities are *objective.* For example, if a coin is fair, then the propensity (probability) of getting a Head when the coin is tossed is 0.5.

Bayesians believe that probability describes the degree of belief in a proposition. Bayesians regard a probability of 0.5 of getting a Head as meaning that there is no reason to expect one outcome over the other. For Bayesian, probability is *subjective*. Different observers have different information and therefore may assign different probabilities to the same propositions. Bayesians deny there is any intrinsic probability associated with physical events. Jaynes [5] systematically destroys any notion, in a highly entertaining discussion, that it makes sense to assign a probability based on the physical properties of the coin.

There are problems with the frequentist approach. For example, what does it mean to say: “*The probability of rain tomorrow is 30%*”? The event is a one-off. It cannot be re-run. So how is it possible to talk about any frequency? Similarly, Frequentists have trouble with Heisenberg’s Uncertainty Principle; Heisenberg applies to unique one-off particle experiments as well as multiple measurements on assemblies of identical particles.

There are many examples where the notion of probability is divorced from frequency. In financial markets, an extensive literature has been developed which uses probabilities to value financial products, but these probabilities do not reflect real propensities (no one knows what the market will do) or even historic frequencies. These probabilities represent a "market view", which according to Economic Theory encompasses (or should) all the information available to the market.

The Bayesian interpretation of probability is at the heart of the Copenhagen Interpretation of Quantum Mechanics.

All calculations are based on the presumption that the results returned by devices on the quantum-classical boundary can be taken at face value. I.e.

$$P(\mathbf{A}) = P(\mathbf{A}|\mathbf{\Psi}) = \text{probability of } \mathbf{A} \text{ given } \mathbf{\Psi}$$

where the state $\mathbf{\Psi}$ is constructed from knowledge of the experimental setup and the history of measurements returned by devices at the quantum-classical boundary.

We shall adopt Bayesian notation without comment.

$$P(\mathbf{A}) = P(\mathbf{A}|\mathbf{I}) = \text{probability that } \mathbf{A} \text{ is true given } \textit{prior information } \mathbf{I}.$$

$$E(\mathbf{A}) = E(\mathbf{A}|\mathbf{I}) = \text{expected value of A given prior information } \mathbf{I}.$$

4.2 Subjective Probability in Quantum Mechanics

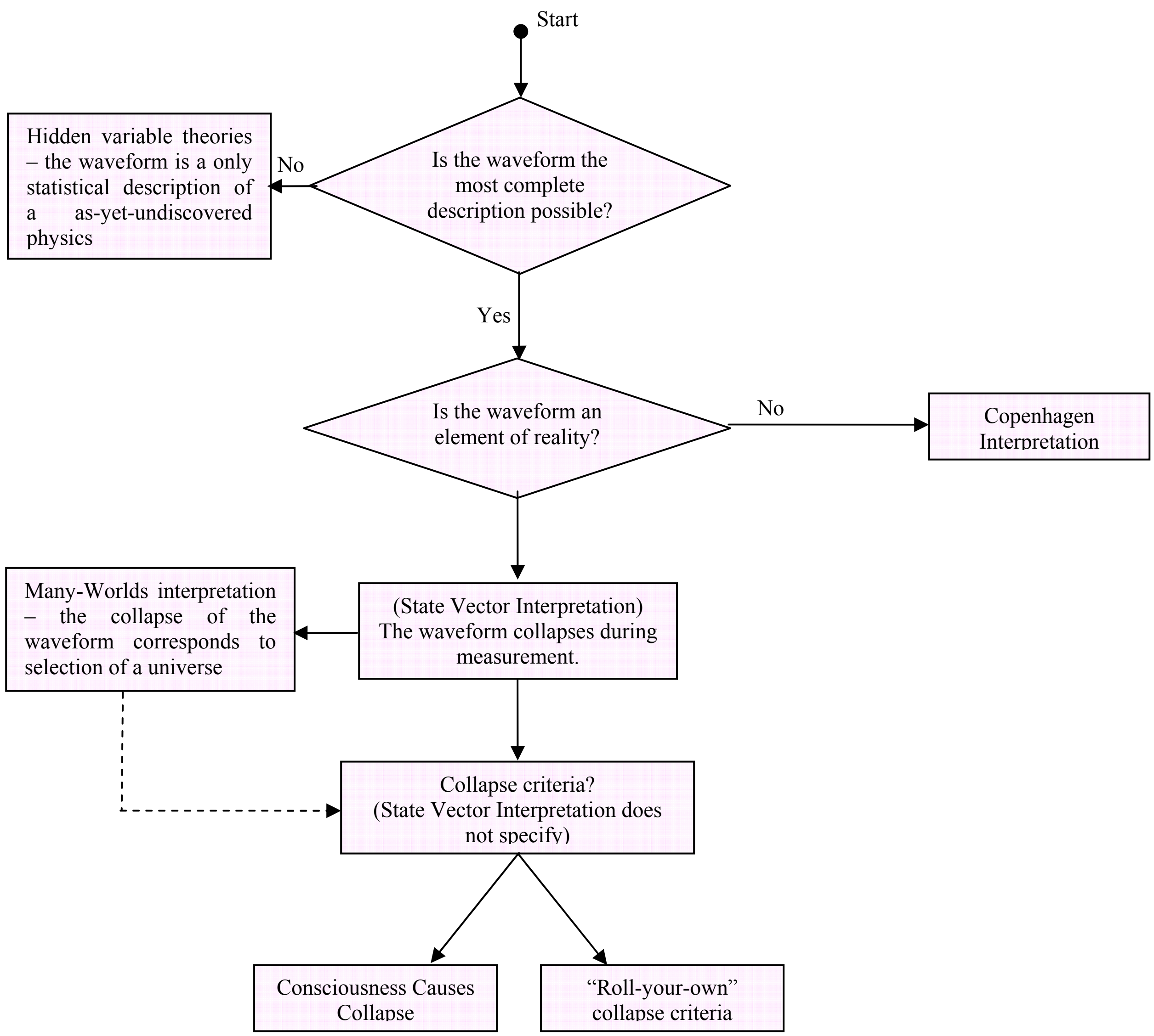
The Bayesian position provides a good fit with the Copenhagen Interpretation of Quantum Mechanics. Quantum Mechanics describes physical systems so it would be expected to produce *objective* probabilities. For example, it would be expected that the probability a photon will be emitted must be independent of the observer.

Paradoxes such as Wigner's Friend clearly show that Quantum Mechanics produces *subjective* probability measures. Different observers will agree only if they have the same relevant information and use the same mental models.

If two observers are calculating the probability of photon emission, then both observers should arrive at the same probabilities for the emission of a photon if both use the same equations and both have the same understanding of the experimental setup. If they use different models (equations), then they may calculate different probabilities. But in a Universe that is only knowable through experiment (measurement) and theory, who is to say who is right and who is wrong? It is only possible to adjust the likelihood of the "correctness" of each model based on existing knowledge (*prior* information) and subsequent events, i.e. the iterative application of subjective Bayesian probability.

5 Which Interpretation?

- The various interpretations arise from the different ways of answering some fundamental questions. See flowchart below.



It seems to me that only the Copenhagen interpretation is viable. It also happens to be one of the few interpretations with underpinings deeply routed in Western philosophy.

Specifically,

- The Many Worlds interpretation is a bit of a cop out. It identifies the sample space of any experiment as “real” by playing semantic games with the meaning of the term. It does not contribute to the understanding of how and when the waveform collapses (= how and when our “universe is selected”). The Many-Worlds interpretation has serious problems with EPR.

- Consciousness Causes Collapse, like all interpretations with a “real” waveform, has trouble with EPR and how to specify exactly what constitutes consciousness.

- The Consciousness Causes Collapse Interpretation arises from failing to select a quantum-classical barrier, tracing the measurement process backwards and finding that a quantum-classical barrier must exist somewhere. The quantum-classical barrier is then chosen to be in the last possible place - the mythical interaction of the quantum world with consciousness.

- Decoherence confuses engineering specification with interpretation.

- Consistent Histories concentrates on applying standard logic and probability theory to a specific choice of sample space (framework). Consistent Histories provides more rigorous versions of some of earlier statements associated with Copenhagen. Unfortunately others aspects of Consistent History, such as a real waveform, render this interpretation non-viable.

- Hidden variable theories cannot be ruled out, but no-one has come close to producing a real alternate to standard Quantum Mechanics. It can be argued that Hidden Variable theories are unscientific since Heisenberg’s Uncertainty Principle means that they can never be experimentally “proved”.

- The Copenhagen interpretation is a Positivist interpretation. Quantum Mechanics is just a probability calculus which can be applied to a specific combination of measuring devices and quantum system. The waveform has no reality; there is no mysterious waveform “collapse”.

- The correspondence between the Copenhagen interpretation and the Bayesian interpretations of classical probability theory is so complete that it is possible to view the Copenhagen interpretation simply as

  (i) Heisenberg's Uncertainty Principle, plus
  (ii) Bayesian interpretation of probability (unreal waveform), plus
  (iii) Positivism (no deeper meaning than that obtained through measurement)

Many of the statements of non-reality and subjective probabilities that some physicists find so disturbing find equivalent statements in modern “classic” probability theory. However the history of classical probability does not bode well for the resolution of the battle between competing interpretations. Classic probability theory is hundreds of years old, but there is still an active debate about which interpretation is correct.

- The “collapse” of a real waveform is the 21st Century equivalent of Luminiferous Ether. Quantum Mechanics is simpler without it. It is responsible for most paradoxes. It generates complications and confusion. To quote Heisenberg, it is *Mist*. It has no place in 21st Century physics.

The Copenhagen Interpretation is pretty much trouble free. Its original formulation is a bit vague and dated in places, but is still the interpretation of choice.

Part B

Measurement

6 Measurement

This part assumes the correctness of the Copenhagen Interpretation outlined in Part A.

6.1 Principles of Measurement

Measurement is not a completely mysterious process. There are quite a few statements that can be made, which are summarized by the Principles M1 – M5 below.

- ***M1***. Principle of Real Measurement.
- ***M2***. Principle of Exact Measurement.
- ***M3***. Principle of Repeatability (a.k.a. Projection Postulate).
- ***M4***. Principle of State.
- ***M5***. Heisenberg's Uncertainty Principle.

These principles have surprisingly consequences, which will be explored later. They are also surprisingly poorly understood.

6.2 Principle of Real Measurement (M1)

M1: *A measurement instantaneously and discontinuously changes the probability distribution associated with some **R**-valued physical quantity.*

Classically a measuring device "targets" a physical property. For example, a police radar gun "targets" the speed of cars and produces a single ***R**-valued* number.

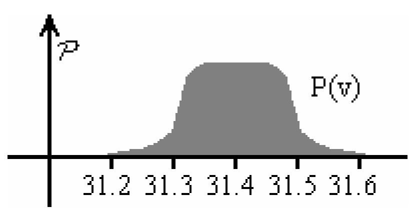


Figure 5.2.1

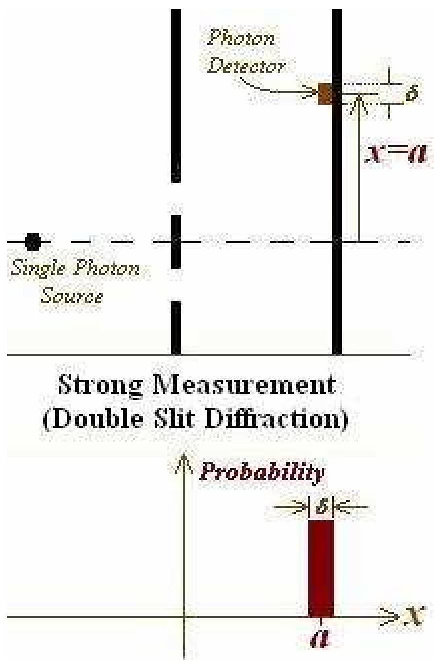
However, the radar gun can only determine the car's speed to some tolerance; the outcome of the measurement is actually a probability distribution. Figure 5.2.1 (above) shows a typical distribution; in this case the digital readout would show a speed of 31.4 k.p.h. (the mean of the probability distribution), and the radar gun would be deemed to be accurate to within 0.2 k.p.h. (the standard deviation of the distribution), slightly more than the resolution of the display 0.1 k.p.h.

An "exact" measurement of physical quantities is generally not possible or even desirable. For example, Heisenberg Uncertainty Principle implies that an "exact" measurement of a car's speed (and momentum) would result in an infinite degree of uncertainty in the car's position; the result would be the random and instantaneous transportation of the car and any occupants to a random location across the universe, almost certainly not in one piece. A radar gun that measured a car's momentum "exactly" would be a very impressive weapon indeed.

6.2.1 Weak and Strong measurements

Definition: A measurement is a strong measurement if the probability density is reduced to zero outside some region [a, b] of R, where [a, b] is not the set of all possible outcomes.

Example of a strong measurement: In diagram 5.2.2 (below), a source emits a single photon in a standard double-slit diffraction mechanism. A photon detector with resolution δ is located on the plane at $x = a$. If a photon is detected then the resulting probability distribution is zero outside the region $[x - \delta, x + \delta]$.



Probability distribution if a photon is detected at x = a.

Figure 5.2.2

Definition: A measurement is a weak measurement that is not a strong measurement.

Example of a weak measurement: All known measurements of time are weak. For example, see the clock construction elsewhere in this document. It can never say that the elapsed time has zero probability of being outside of $[t_0, t_1]$ for some t_0, t_1.

6.2.2 M1 – Corrections to Common Misunderstandings

These arise from misinterpretation of the formalism of Quantum Mechanics.

The Principle of Real Measurement does ***not*** *say "a measurement of a quantity results is a* **R**-*valued number".*

6.2.3 Why R?

Why are physical quantities ***R***-valued? There are a number of partial rationalisations. For example, Kant pointed out that humans reason with mental models. Humans use Natural numbers, and Real numbers are a natural extension of Natural numbers.

But ultimately, as Feynman might say, that's just the way the Universe is.

6.3 Principle of Exact Measurement (M2)

M2: *Measurements at the Classical-Quantum boundary are always 100% accurate*.

This is sometimes unfairly criticised as tautological**:** a measuring device is always 100% accurate because it measures whatever it measures! If Bohr is correct and there is no deeper understanding of the measurement process, how is it possible to claim a measurement is in error?

6.3.1 Quantum Measurement Devices

In truth, no principle could have been more clumsily worded. The principle cannot be fully understood without reference to the Kantian / Positivist philosophy that underpins it. Bohr (Kant) argues that observers interact with the world through measuring devices (senses). Rational observers have little choice but to accept the results of their measurements (what their senses tell them).

The measuring devices used by an observer defines the Kantian boundary between the external world and the observer. The boundary is logical defined by the flow of information to the observer; it is not physical barrier. Once a measurement has been made, the information becomes "classical" – it becomes part of the observer's experience and it no longer matters what was on the other side of measuring device.

Traditionally the Kantian boundary is called the Classical-Quantum boundary – a truly terrible name.

It is the belief in the correctness of the measuring devices (senses) on the Kantian boundary that converts probability into "fact", to the extent that anything is fact. The statement "*The sky is blue*" is fact because we accept what our senses tell us.

6.3.2 The Measurement Problem - Revisited

Copenhagen resolves "The Measurement Problem" by insisting that after a (strong) measurement a system is definitely in one or other of the possible outcomes – the result from the measuring device is accepted at face value. For example, an eye determines whether the sky is blue or not, then after a measurement the system can be in the state

| sky is blue >

or in the state

|sky is not blue >

but is not in the superimposed state

$$p \,|\, \textbf{sky is blue} > + (1 - p)\, |\textbf{sky is not blue} > \quad p \neq 0,1$$

6.3.3 M2 – Corrections to Common Misunderstandings

The Principle of Exact Measurement does ***not*** say that the standard deviation of the waveform after a measurement must be zero. All measuring devices have a "resolution"; the waveform is only constrained to a limited set of values.

For the same reason, the Principle of Exact Measurement does ***not*** require the system to be in an eigenstate after measurement.

6.3.4 Classical Measurement Devices

Classically measurements are assumed to include "error". The speed displayed by a police speed gun can definitely be "wrong" - prosecutions for speeding have failed for exactly that reason. So how is it possible to reconcile **M3** with the classical mechanics?

The radar gun does indeed measure something with 100% accuracy: Electric currents in circuits that are affected by Doppler shifted radiation reflected by the car, and possibly other objects, which in turn is affected by atmospheric conditions, etc. The figure returned by the radar gun is a function of many variables, over which there may be limited control. Macroscopic measuring devices are highly engineered, and as Bohr pointed out, it is our knowledge of engineering which allows us to construct them and understand how they work. We are confident that a measurement of some electric current is also an accurate measurement of car speed if the final displayed speed returned by the gun corresponds closely with what we believe should be the "right" answer.

We shall return to the question of determining what a measuring device measures later.

6.4 Principle of Repeatability (M3) – Projection Postulate

M3: "*From physical continuity, if we make a second measurement of the same dynamical variable immediately after the first, the result of the second measurement must be the same as that of the first"* - Dirac, 1947.

At first no principle could be simpler, but surprisingly it implicitly imports a significant number of assumptions. This principle is sufficiently complex that discussion is postponed until later in the document.

6.5 Principle of State (M4)

How much knowledge is required to completely describe a quantum system?

Quantum systems are surprisingly easy to describe. Understanding what a Hydrogen atom will do does not require an understanding of what happened in the past. The hydrogen atom can be described by a limited set of quantum numbers.

Mathematically any quantum system is completely described by a state vector $\mathbf{\Psi}$ and Hamiltonian $\mathbf{H}$. Knowledge of the previous history of the waveform is irrelevant to the calculation of the probabilities of any experimental outcomes. Once the system is known to be in a state $\mathbf{\Psi}$ then the future state of the system (before any measurement) is given by $\mathbf{\Psi}(t) = \mathbf{\Psi}_0 e^{-\frac{i}{\hbar}\mathbf{H}t}$. The Hamiltonian $\mathbf{H}$ characterises the system but does not contain any information about the current state of the system.

6.6 Heisenberg's Uncertainty Principle (M5)

M5: *Heisenberg's Uncertainty Principle famously states that for any particle,* $\sigma_x \sigma_p \geq \frac{\hbar}{2}$ *where* σ_x *and* σ_p *are the standard deviations (uncertainty) of the probability distribution for the particle's position x and its momentum p.*

6.6.1 Common Misunderstandings

"If the velocity of the electron is at first known, and the position then exactly measured, the position of the electron for times previous to the position measurement may be calculated. For these past times, δpδq is smaller than the usual bound." - Heisenberg 1930, p. 15.

Does Heisenberg apply to single particles or only ensembles?

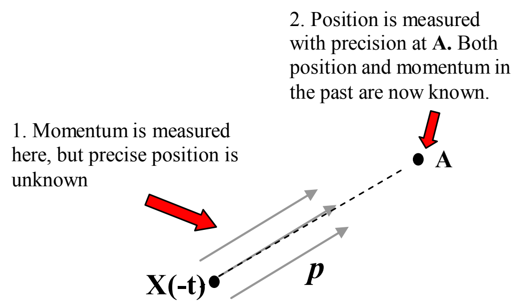
A number of authors have suggested that Heisenberg's Uncertainty Principle can only be defined in terms of particle ensembles. This is not so; the motivation behind this statement seems to be a particular Frequentist interpretation of probability.

Heisenberg applies to single particles and not just ensembles.

Does Heisenberg's Uncertainty Principle hold for the part?

"*The uncertainty relation does not hold for the past." - Heisenberg, 1930.*

The following question is commonly asked by physics students*:* Suppose the momentum of an electron is precisely measured, and then the particle is found at **A**, why is it not possible to project backward to determine the position at any time in the past? In particular, why is it not possible to project the electron's position back to the time when the momentum was measured? Wouldn't that be a violation of Heisenberg's Uncertainty Principle?

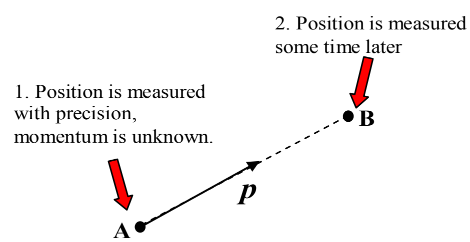


The location of the particle *in the past* could, in theory, be determined by the formula.

$$\vec{x}(-t) = \vec{x}_A + \frac{(-t)}{m}\vec{p}$$

Heisenberg resolves this paradox by simply stating that the uncertainty relations simply do not "does not hold for the past".

A second version of this is: Suppose the position of an electron is found at **A** and then some time δt later it was found at **B**. Why is it not possible to calculate the momentum ***p*** of the particle when it was at **A** from the known positions and time interval δt?



The momentum of the particle *in the past* could, in theory, be determined by the formula.

$$\vec{p} = \frac{m.(\vec{x}_A - \vec{x}_B)}{\delta t}$$

Again, this is not a violation since Heisenberg's uncertainty relations "do not hold for the past".

Bohr, however, would have been far more savage. Something is known only if it is measured. It is falacious to project backward in time and say, ***if*** we had measured the momentum or position at a certain time we would have got such-and-such a result for at least 2 reasons:

(1) The act of measurement would have changed the future evolution of the system so there is no guarantee the system would have evolved into it's later known state if the measurement *had* been made.

(2) Bell (and Von Neumann) would point out that if a measurement of, say, position is made, then the momentum is not only not known, it is not defined. It is simply wrong to base an argument on the assumption that it is.

6.6.2 Robertson's Inequality- Generalised Heisenberg

In 1929, Howard Robertson produced a generalized version of Heisenberg's Uncertainty Principle. If **A** and **B** are measurement operators, then for any state **Ψ**

$$\sigma_{\mathbf{A},\Psi}.\sigma_{\mathbf{B},\Psi} \geq \frac{1}{2}\left|E_{\Psi}([\mathbf{A},\mathbf{B}])\right|$$

where $\sigma_{\mathbf{A},\Psi}$ is the standard deviation of the probability distribution of **A,** $\sigma_{\mathbf{B},\Psi}$ is the standard deviation of the probability distribution of **B,** $E_{\Psi}(\mathbf{X})$ is the expected value of the random variable **X**. (See Appendix B for proof)

Robertson's Inequality reduces to Heisenberg when **A** and **B** are momentum and position operators.

6.6.3 The Bohr-Einstein Debates

Einstein was unhappy with many aspects of Quantum Mechanics. During the 5th and 6th Solvay Conferences in 1927 and 1930 respectively, Einstein challenged Bohr with a number of thought experiments aimed at showing that HUP could be circumvented. Einstein failed, but succeeded in showing that Heisenberg's Uncertainty Principle applied even to classical mechanics.

6.6.4 Thought Experiment 1 – Double slit diffraction

This thought experiment is a version of the standard double slit diffraction for a single particle. The construction was intended to allow the experimenter to determine which slit in the middle screen the particle goes through.

A particle beam is projected onto the left most screen, the intensity of the beam is sufficiently low that only a single particle will be present in the system at any time. A circular waveform radiated from the leftmost screen and onto the double slits in the middle screen. Diffraction occurs and the particle is detected when it strikes the rightmost screen. As more and more particles strike the rightmost screen, the standard double-slit diffraction pattern is built up.

Einstein argued that the particle momentum could be determined by measuring the momentum imparted to the leftmost as the particle "changed direction" going through the screen. If the particle position and momentum are known simultaneously (as it goes through the middle screen), then Heisenberg's Uncertainty principle cannot hold.

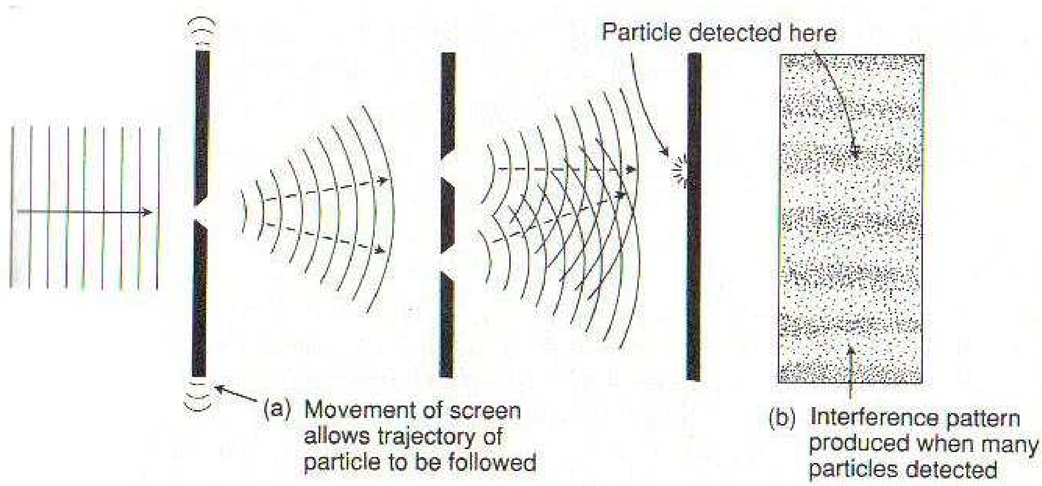


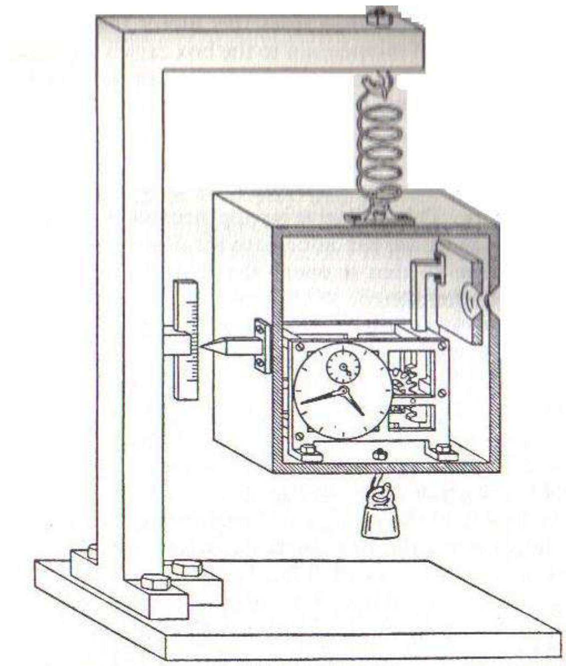
Figure 5.6.44
Bohr-Einstein Diffraction

Bohr pointed out that the leftmost screen would itself be subject to Heisenberg's Uncertainty Principle; a "successive" measurement of momentum imparted to the screen would destroy the double-slit interference pattern on the rightmost screen.

6.6.5 Thought Experiment 2 – The photon box

The photon box consists of a clock in a box with a small hole in one side. A shutter covers the hole. The inside of the box is flooded with photons, and weighed. At a pre-determined time the shutter is opened, a single photon is allowed to escape and the shutter is then closed. The box is weighed again. The energy of the photon is equal to the change in weight of the box multiplied by c^2.

Einstein suggested that the device could provide precise information about the emitted photon's energy at a specific time, contrary to the energy-time version of HUP.*



Einstein's photon box.

Bohr's responded that the box has to be illuminated to "see" the weight shown on the scale, which imparts random momentum to the box. I.e. it jumps around uncontrollably. It is possible to measure the *average* position to arbitrary precision by waiting a sufficiently long period of time for the pointer to settle down, but the momentum uncertainty remains. Now Bohr's *coup de grâce*. According to Einstein's own General Theory of Relativity, the movement of the clock in the gravitational field affects the rate at which the clock runs. The uncertainty in the box's momentum produces an uncertainty in the time of the shutter opening of a magnitude in agreement with HUP.

* Strictly speaking, no Robertson style uncertainty relationship exists between energy and time (there is no time operator) but an analogous inequality can be formulated.

6.6.6 Additional Uncertainty

Suppose **A** and **B** are conjugate quantities, then the results of measurements can be written.

$$\mathbf{A} = \langle\mathbf{A}\rangle + \delta\mathbf{A}$$

$$\mathbf{B} = \langle\mathbf{B}\rangle + \delta\mathbf{B}$$

where $\langle\mathbf{A}\rangle$ and $\langle\mathbf{B}\rangle$ are the expected values of **A** and **B**.

$$\sigma_{\mathbf{A}}^2 = \sigma_{\langle\mathbf{A}\rangle+\delta\mathbf{A}}^2 = \sigma_{\langle\mathbf{A}\rangle}^2 + Cov(\langle\mathbf{A}\rangle, \delta\mathbf{A}) + \sigma_{\delta\mathbf{A}}^2$$

By definition, $\sigma_{\langle\mathbf{A}\rangle} = \mathbf{0}$, so $Cov(\langle\mathbf{A}\rangle, \delta\mathbf{A}) = 0$ and $\sigma_{\mathbf{A}}^2 = \sigma_{\delta\mathbf{A}}^2$

Similarly, $\sigma_{\mathbf{B}}^2 = \sigma_{\delta\mathbf{B}}^2$

A and B a re conjugate quantities os $\sigma_{A.} \sigma_B > k$ for some $k > 0$

$$\Rightarrow \sigma_{\delta A} \cdot \sigma_{\delta A} > k$$

I.e. the difference between the expected value of a quantity and the measured value obeys the same uncertainty relationship as the original conjugate quantities.

6.7 The Correspondence Principle

The Correspondence Principle, originally due to Bohr, requires that "*for macroscopic objects*" Quantum Mechanics must make predictions which closely agree with classical mechanics. The Correspondence Principle is not a principle in the same way as Principles *M1 – M6*, rather it is a test for how "good" a theory is: If a theory does not satisfy the Correspondence Principle, then it is not a good fit with the observed classical world and therefore not a good theory.

6.8 The Calibration Problem

Bohr's view was that measuring devices are "essentially classical" in the sense and the examination of their macroscopic structure determines what they measure. Bohr's approach fails completely when considering abstract properties such as quark colour; there is no way to relate quark colour to any macroscopic measuring device, yet colour is clearly a valid and useful construct. Bohr's approach is useful, but limited. A better approach is needed to handle the ideal measurements used in theoretical calculations.

Part C

The

London (Ticker Tape) Interpretation

of

Quantum Mechanics

7 The Ticker-Tape Interpretation of Quantum Mechanics

Abstract: In recent years there has been a move away from the Copenhagen Interpretation towards alternative interpretations of Quantum Mechanics. There has also been an acknowledgement that there is no definitive version of the Copenhagen Interpretation because the originators, Bohr, Heisenberg et. al., did not agree over all aspects of the interpretation. This paper revisits the philosophical approach taken by Bohr and Heisenberg. The result is a new interpretation, named the Ticker-Tape Interpretation, which is closely related to Copenhagen. The interpretation leads to some conjectures.

7.1 Philosophical Underpinnings

This paper adopts a world view that is essentially Kantian. The Kantian world view consists of an external world that is perceived through the sensors of an agent; the agent builds mental models of the external world but cannot know its true nature.

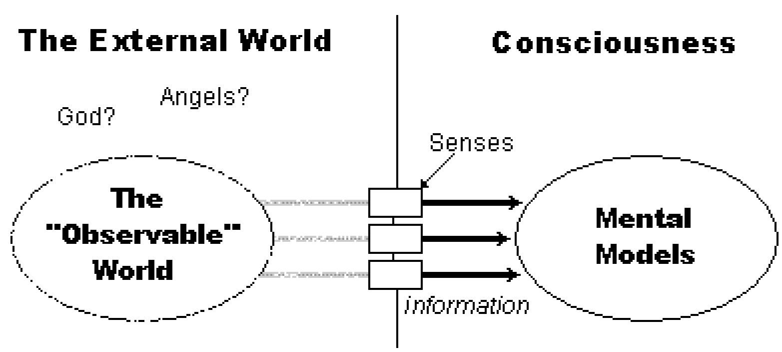


Figure 1.

Kantian World View (circa 1785).

Copenhagen clarifies aspects of the Kantian world view. (It has been argued that the Copenhagen Interpretation is Positivist, however subtle arguments over philosophical classifications are beyond the scope of this paper) Quantum Mechanics replaces the vague idea of sensory input with that of measurement and precise mathematical description. The idea of a conscious agent is replaced by intelligence, which need not be human (although some commentators would dispute this). It could, for example, be a robot.

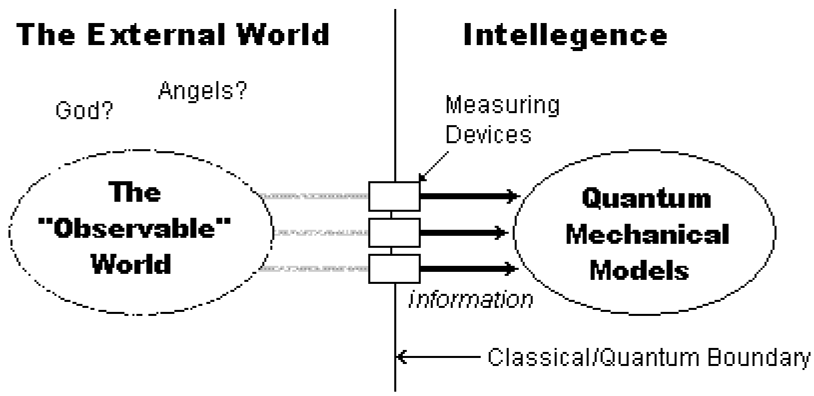


Figure 2.

Copenhagen World View (circa 1928).

The Copenhagen Interpretation acknowledges that our mental models may be incomplete; it may be necessary to apply a wave model in some situations, and apply a particle model in another (Complementary Principle). Realism, on the other hand, is rooted in the belief that the world is as it appears, and seeks to preserve macroscopic models, possibly beyond their domain of applicability.

7.2 Measurement

Measurements are regarded as "elements of reality", although no definition of what this means is given. The observer is logically separate from the measurement itself. A measurement discontinuously changes the probability distribution associated with a "physically significant" random variable.

A measurement can be represented as a pair ($\mathbf{X}$, φ) where $\mathbf{X}$ represents the physical quantity measured (more accurately the device used to measure it) and φ is a probability density function.

Definition: A device that produces a single representative number as its output is called a "strong" measuring device, and the measurements it produces are called "strong" measurements. The notation $\mathbf{X} = x$ is sometimes used to mean a strong measurement of the physical quantity denoted by $\mathbf{X}$ has been made and reported as x.

Definition: A history is a sequence of measurements and denoted $\mathcal{H} = (\mathbf{m}_0, \mathbf{m}_1, \mathbf{m}_2, ..., \mathbf{m}_n)$ where $\mathbf{m}_i$ are measurements. If all measurements are "strong", the history will look like ($\mathbf{X}_0 = x_0$, $\mathbf{X}_1 = x_1$, $\mathbf{X}_2 = x_2$, ..., $\mathbf{X}_n = x_n$).

7.3 The Equivalence Principle

There is no transition from the Quantum world to the Classical world. The difference between measurement in the Classical world and measurement in the Quantum world is a matter of interpretation.

Quantum Mechanics is a mental model; the waveform and any measurement operator **A** are mental constructions, built on top of the information gathered from “raw” measurements. Classical mechanics is also a mental model (since we know it is not “true”, it cannot be otherwise); typically Classical Mechanics deals with <**A**> and regards the difference from <**A**> to be “error”.

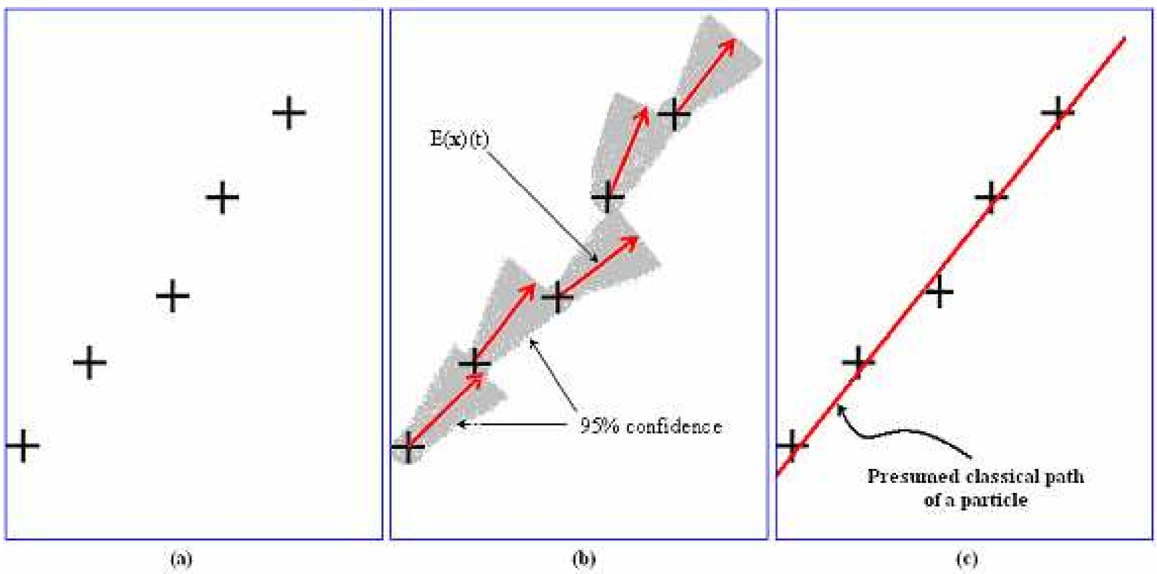


Figure 6
Measurements and Mental Models.

In the diagram above, the left pane shows the “raw” position measurements of a particle moving diagonally from left to right is shown. The central pane *adds* the intellectual machinery of Quantum Mechanics; the *calculated* 95% confidence intervals are shown in gray. The right pane *adds* the presumed classical path.

In Quantum Mechanics, the uncertainty in the position of a particle created by Heisenberg Uncertainty Principle is viewed as intrinsic to the system; the measurement themselves are taken at face value (*The Principle of Exact Measurement*). In Classical Mechanics, the “error” (uncertainty) in the position of the particle has a multitude of sources, typically related to the construction of the measuring devices and lack of knowledge of initial conditions, but there is a presumption that if these influences could be eradicated, exact measurements would be possible.

7.4 Mach Devices

Bohr expressed the opinion that measuring devices are essentially classical, and that a description of reality required a-prior understanding of classical quantities such as position and momentum [1]. There are difficulties with this approach. For example, the approach requires a knowledge of classical mechanics before Quantum Mechanical measurements can be understood, yet Quantum Mechanics is presumed to be more fundamental than classical physics. It is also not easy to apply Bohr's vision of essentially classical measuring devices to abstract concepts such as QCD colour.

Rather than follow Bohr or Kant, we take a slightly different view of measurement.

Definition: A measuring device is a Mach device with respect to an observer if the following applies:

(i) The device produces a stream of measurements that are recorded on a ticker-tape (or equivalent) accessible to the observer. Measurements are recorded in the order they are made.

(ii) The device's internal structure is unknown. It is a "black box". There is no *a-prior information* about what the numbers it produces mean.

(iii) The observer does not have access to a clock.

Figure 7

The output of a Mach device viewed as a "ticker tape".

A Mach device does not need to be a strong measuring device, but for convenience we shall almost always use strong measurements from this point on.

7.5 Principle of Relativity (Mach's Principle)

The output from a single Mach device is meaningless.

This is a generalisation of Mach's Principle. Mach's statement of the Principle of Relativity famously influenced Einstein but the principle itself dates back at least to Galileo. Ernst Mach argued that it would be meaningless to talk about the motion of a single particle in an empty Universe. All motion is relative. In fact, all measurement is relative. If there is no context, a measurement stream becomes a meaningless stream of numbers.

The Principle of Relativity is closely related to The Calibration Problem: What does a measuring device measure?

7.5.1 Mach Banishes Determinism

Suppose $\boldsymbol{X} = f(t)$ is a classical quantity, and **X** is a measuring devices that faithfully returns ***X***. If **X** is a Mach device with respect to an observer, it is not possible for that observer to determine what the device measures from the measurement history. Why? There is no way to calibrate the device.

Suppose we construct a second device **Y** whose output is related to the first by

$$\mathbf{Y} = \zeta\,(\mathbf{X}(t))$$

The second device is sealed, mixed up with the first and given to the naive observer so they both become Mach devices. Which measures the "fundamental" quantity? **X** and **Y**? **X**? **Y**? In fact, we can build the device **Y** so it returns any measurement profile we like.

If a measuring device qualifies as a Mach device ***except*** *that it is known that it measures a quantity* **X** *where* **X** *= f(t) for some function f , then the output from that device is meaningless.*

7.6 State

The test of a good theory is whether it can make accurate predictions. In the case of Quantum Mechanics, the obvious question is: How much history is necessary before an observer can make accurate predictions? The answer cannot be that the observer must know the entire history of a system since the beginning of time since that information will never be available. So what are the alternatives? One possibility is to only count the last N measurements for some N, perhaps giving the more recent measurements more "weight". But how do we chose N? How do we "weight" different measurements? The issue of numeric stability also raises its head. One class of measuring devices however provides us with an easy win.

Conjugate Devices

Definition: **A** and **B** are conjugate devices if a measurement of **A** "wipes out" the measurement of **B**., and a measurement of **B** "wipes out" the measurement of **A**. More precisely

$$P(\mathbf{x} \mid (\mathbf{A}=a, \mathbf{B}=b)) = P(\mathbf{x} \mid \mathbf{B}=b) \text{ for all } \mathbf{x}, a, b$$

and

$$P(\mathbf{x} \mid (\mathbf{B}=b, \mathbf{A}=a)) = P(\mathbf{x} \mid \mathbf{A}=a) \text{ for all } \mathbf{x}, b, a$$

where the probabilities P(X|Y) are derived via Bayesian analysis of a large number of previous measurements.

Definition: The minimal sub-history $\mathbf{\Psi}$ of $\mathcal{H}$ such that $P(\mathbf{x} \mid \mathbf{\Psi}) = P(\mathbf{x} \mid \mathcal{H})$ for all $\mathbf{x}$ is called the state of the system.

7.6.1 Principle of State

*If the **A** and **B** are conjugate measurement devices, the state consists only of the last measurement made by one of the devices*

8 Feyman's Rules

We follow the argument put forward by Ariel Caticha[2].

Definition: If $\mathcal{H}_A = (\mathbf{m}_1,..., \mathbf{m}_k)$ and $\mathcal{H}_B = (\mathbf{m}_{k+1}, ..., \mathbf{m}_{k+m})$ are histories, then

$$\mathcal{H}_A \wedge \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_k, \mathbf{m}_{k+1}, ..., \mathbf{m}_{k+m})$$

The history $\mathcal{H}_A$ must "follow" the history $\mathcal{H}_B$ and not overlap in time. The operator $\wedge$ is read as "and".

Definition: Let $\mathcal{H}_A$ be a history, then $\mathcal{H}_B$ is an alternative history to $\mathcal{H}_A$ if $\mathcal{H}_B$ is the same as $\mathcal{H}_A$ except that some of the measurements, other than the initial and final measurements, have different values.

Definition: If $\mathcal{H}_A = (\mathbf{m}_1, \ldots, \mathbf{m}_{Ai}, ..., \mathbf{m}_n)$ and $\mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_{Bi}, ..., \mathbf{m}_n)$ are alternative histories which differ in the value of the i^{th} measurement, then

$$\mathcal{H}_A \vee \mathcal{H}_B = (\mathbf{m}_1, \ldots, \mathbf{m}_{Ai} + \mathbf{m}_{Bi}, ..., \mathbf{m}_n)$$

The operator $\wedge$ is read as "or".

The operators $\wedge$ and $\vee$ obey the following relations:

$$a \vee b = b \vee a$$

$$(a \vee b) \vee c = a \vee (b \vee c)$$

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

If $\mathcal{H}$ = set of possible histories for a system, the any representation (Ψ, +, x) with Ψ: $\mathcal{H} \rightarrow \Omega$ and

$$\Psi(a \vee b) = \Psi(a) + \Psi(b)$$

$$\Psi(a \wedge b) = \Psi(a) \times \Psi(b)$$

would need to carry these properties across.

Caticha[2] shows that for the general solution for complex valued representations + and x are the standard addition and multiplication operators.

9 The Formalism of Quantum Mechanics.

The representation of measurements as pairs (**X**, φ) and system states as sequences of measurement (histories) is useful from a conceptual perspective but sorely lacks the computational power that comes with a "good" representation.

Definition: We expand the definition of state to include *any* representation $\mathbf{\Psi}$ such that P(**x** | $\mathbf{\Psi}$) = P(**x** | $\mathcal{H}$) for all **x** where $\mathcal{H}$ is the known history of the system.

9.1 Principle of Relativity (No Preferred Frame of Reference)

If $\mathbf{\Psi}$ *is a representation of state, then the rule for the calculation of* P((**A**,φ)|$\mathbf{\Psi}$) *should be the same as* P((**B**,χ)|$\mathbf{\Psi}$) *irrespective of the state* $\mathbf{\Psi}$*, or devices* **A** *or* **B**.

Author's Note

Here the concept of "frame of reference" is used loosely to mean "set of measuring devices".

The goal of the remainder of the document is to develop the standard formalism of Quantum Mechanics.

Plan of Attack: (1) The formalism will focus on the expanded concept of state rather than histories or measurements since state is not intrinsically tied to any particular set of measurements (frame of reference). For example, suppose we construct a device **C** from **A** (or a pre-cursor to **A**) such that the measurement (**C**=c) means that there is a 20% chance of **A**=a_1 and a 80% chance of a measurement of **A**=a_2. The abstract representation $\mathbf{\Psi}$ relieves us of have to decide whether to represent the system state as (**A**, (P(a_1)=0.2, P(a_2)=0.8) or (**C**=c)

(2) The formalism of Quantum formalism will be developed using commuting measuring devices. The Principle of Relativity (No Preferred Frame of reference) will then be used to justify the application of the same rules to conjugate devices, resulting in the algebraic extension of the state space.

(3) We restrict our arguments to systems with stable states and transition probabilities.

Definition: The state $\mathbf{\Psi} = \lambda\mathbf{\Psi}_1 + \mu\mathbf{\Psi}_2$ is the state with the following property.

$$P(\mathbf{\Phi} \,|\, \lambda\mathbf{\Psi}_1 + \mu\mathbf{\Psi}_2) = \lambda\, P(\mathbf{\Phi} \,|\, \mathbf{\Psi}_1) + \mu\, P(\mathbf{\Phi} \,|\, \mathbf{\Psi}_2) \text{ for all measurements } \mathbf{\Phi} \text{ and } \mu, \lambda \in \Re$$

Note that P: $\mathcal{S} \times \mathcal{S} \rightarrow [0,1]$ where $\mathcal{S}$ is the system state space so we must also consider the meaning of $P(\lambda\mathbf{\Phi}_1 + \mu\mathbf{\Phi}_2 \,|\, \mathbf{\Psi})$.

Recall that $\Psi(a \vee b) = \Psi(a) + \Psi(b)$ so

$$P(\lambda\mathbf{\Phi}_1 + \mu\mathbf{\Phi}_2 \,|\, \mathbf{\Psi}) = P(\lambda\mathbf{\Phi}_1 | \mathbf{\Psi}) + P(\mu\, \mathbf{\Phi}_2 \,|\, \mathbf{\Psi})$$

The interpretation of $\lambda\mathbf{\Phi}$ from earlier yields

$$P(\lambda\mathbf{\Phi}_1 + \mu\mathbf{\Phi}_2 \,|\, \mathbf{\Psi}) = \lambda P(\mathbf{\Phi}_1 | \mathbf{\Psi}) + \mu\, P(\mathbf{\Phi}_2 \,|\, \mathbf{\Psi})$$

P: $\mathcal{S} \times \mathcal{S} \rightarrow [0,1]$ is a bi-linear mapping on the state space and can therefore be has a standard representation as a linear operator. It therefore has a representation as a matrix (given a suitable basis) and inner product $<.|.>$.

Suppose we have a system is the state $\mathbf{\Psi} = \lambda\mathbf{\Psi}_1 + \mu\mathbf{\Psi}_2$, and we want to know the expected value of a measurement of a quantity **Y**. The quantity **Y** can take on 2 values: y_1 (in which case the state of the system is $\mathbf{\Phi}_1$) or y_2 (in which case the state of the system is $\mathbf{\Phi}_2$).

The expected value $<\mathbf{Y}>$ is calculated as follows:

$$\begin{aligned} <\mathbf{Y}> &= y_1\, P(\mathbf{\Phi}_1 | \mathbf{\Psi}) + y_2\, P(\mathbf{\Phi}_2 | \mathbf{\Psi}) \\ &= y_1 \lambda\, P(\mathbf{\Phi}_1 \,|\, \mathbf{\Psi}_1) + y_2 \mu\, P(\mathbf{\Phi}_1 \,|\, \mathbf{\Psi}_2) + y_1 \lambda\, P(\mathbf{\Phi}_2 \,|\, \mathbf{\Psi}_1) + y_2 \mu\, P(\mathbf{\Phi}_2 \,|\, \mathbf{\Psi}_2) \\ &= y_1 \lambda <\mathbf{\Phi}_1 \,|\, \mathbf{\Psi}_1> + y_2 \mu <\mathbf{\Phi}_1 \,|\, \mathbf{\Psi}_2> + y_1 \lambda <\mathbf{\Phi}_2 \,|\, \mathbf{\Psi}_1> + y_2 \mu <\mathbf{\Phi}_2 \,|\, \mathbf{\Psi}_2> \end{aligned}$$

The last line is the standard expansion shown in Quantum Mechanics textbooks, except of course that coefficients and inner products are real valued.

9.2 Algebraic Extension for Conjugate Devices

Suppose we have two conjugate Mach devices, denoted **A** and **B**. Each device can return one of two values, designated ↑ and ↓ (borrowed from spin, the archetypical quantum property). There are one 4 possible states; the table below shows some of the histories that map onto those states.

State	Histories
$\mathbf{\Psi_{A,\uparrow}}$	(…,**B** = ↓,**A** = ↑),(…,**B** = ↓,**A** = ↑),(**B** = ↑,**A** = ↑),(**B** = ↓,**A** = ↑),(**A** = ↑)
$\mathbf{\Psi_{A,\downarrow}}$	(…,**B** = ↑,**A** = ↓),(…,**B** = ↓,**A** = ↓),(**B** = ↑,**A** = ↓),(**B** = ↓,**A** = ↓),(**A** = ↓)
$\mathbf{\Psi_{B,\uparrow}}$	(…,**A** = ↑,**B** = ↑),(…,**B** = ↓,**B** = ↑),(**A** = ↑,**B** = ↑),(**B** = ↓,**B** = ↑),(**B** = ↑)
$\mathbf{\Psi_{B,\downarrow}}$	(…,**A** = ↑,**B** = ↓),(…,**B** = ↓,**B** = ↓),(**A** = ↑,**B** = ↓),(**B** = ↓,**B** = ↓), (**B** = ↓)

Figure 9.

State Histories Table

The transition probabilities are extracted by observation using standard statistical techniques and shown in the table below.

State (history)	Probability			
	(**A** = ↑)	(**A** = ↓)	(**B** = ↑)	(**B** = ↓)
$\mathbf{\Psi_{A,\uparrow}}$	1	0	0.5	0.5
$\mathbf{\Psi_{A,\downarrow}}$	0	1	0.5	0.5
$\mathbf{\Psi_{B,\uparrow}}$	0.5	0.5	1	0
$\mathbf{\Psi_{B,\downarrow}}$	0.5	0.5	0	1

Figure 10

Transition Probabilities

where $\mathbf{\Gamma}$ is a bi-linear operator; which is the one of the standard postulates for Quantum Mechanics.

Note that this result only holds if $\mathbf{m_1}$ and $\mathbf{m_2}$ are measurements made with the same measurement device.

If A and B are conjugate devices, **A** "wipes out" any knowledge of **B**, so the state space is spanned by $\{\mathbf{\Psi_{A,\uparrow}}, \mathbf{\Psi_{A,\downarrow}}\}$. It follows that

$$\mathbf{\Psi_{B,\uparrow}} = a.\mathbf{\Psi_{A,\uparrow}} + b.\mathbf{\Psi_{A,\downarrow}} \qquad \text{for some } a,b$$

$$\mathbf{\Psi}_{\mathbf{B},\downarrow} = c.\mathbf{\Psi}_{\mathbf{A},\uparrow} + d.\mathbf{\Psi}_{\mathbf{A},\downarrow} \qquad \text{for some } c,d$$

where

$$P(\mathbf{\Psi}_{\mathbf{A},\uparrow} \mid \mathbf{\Psi}_{\mathbf{B},\uparrow}) = 0.5$$

$$P(\mathbf{\Psi}_{\mathbf{A},\uparrow} \mid \mathbf{\Psi}_{\mathbf{B},\downarrow}) = 0.5$$

$$P(\mathbf{\Psi}_{\mathbf{A},\downarrow} \mid \mathbf{\Psi}_{\mathbf{B},\uparrow}) = 0.5$$

$$P(\mathbf{\Psi}_{\mathbf{A},\downarrow} \mid \mathbf{\Psi}_{\mathbf{B},\downarrow}) = 0.5$$

$$P(\mathbf{\Psi}_{\mathbf{B},\uparrow} \mid \mathbf{\Psi}_{\mathbf{B},\downarrow}) = 1$$

$$P(\mathbf{\Psi}_{\mathbf{B},\uparrow} \mid \mathbf{\Psi}_{\mathbf{B},\downarrow}) = 0$$

Solving (after choosing) yields:

$$\mathbf{\Psi}_{\mathbf{A},\uparrow} = \begin{bmatrix}1\\0\end{bmatrix}, \mathbf{\Psi}_{\mathbf{A},\downarrow} = \begin{bmatrix}0\\1\end{bmatrix}, \mathbf{\Psi}_{\mathbf{B},\uparrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}, \mathbf{\Psi}_{\mathbf{B},\downarrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

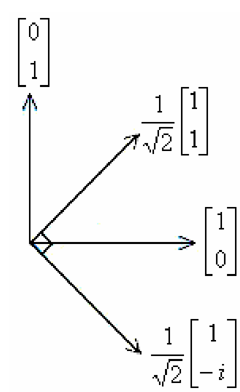


Figure 11

Geometric Interpretation of state space

If a 3rd 2-valued conjugate device **C** is introduced, solving the resulting equations yields.

$$\mathbf{\Psi}_{C,\uparrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix},\ \mathbf{\Psi}_{C,\downarrow} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -i\end{bmatrix}$$

The measurement operators are found to be

$$\mathbf{L}_x = \hbar/2.\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ \mathbf{L}_y = \hbar/2.\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\ \mathbf{L}_z = \hbar/2.\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

If we add a 4^{th} 2-valued quantity conjugate, then solving yields

$$\begin{bmatrix}1\\ 0\end{bmatrix},\ \begin{bmatrix}0\\ 1\end{bmatrix},\ \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \pm i\end{bmatrix},\ \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \pm j\end{bmatrix},\ \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \pm k\end{bmatrix}$$

where *i, j, k* are quarternions. The corresponding operators are

$$\mathbf{K}_0 = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\ \mathbf{K}_1 = \begin{bmatrix}0 & -i.\sigma_x\\ i.\sigma_x & 0\end{bmatrix},\ \mathbf{K}_2 = \begin{bmatrix}0 & -i.\sigma_y\\ i.\sigma_y & 0\end{bmatrix},\ \mathbf{K}_3 = \begin{bmatrix}0 & -i.\sigma_z\\ i.\sigma_z & 0\end{bmatrix}$$

which are recognizable as variants of the Dirac's matrices.

10 The First Ticker-Tape Conjecture

The algebraic relationships between operators define "what we are measuring". In particular, the commutator relationships can be used to classify types of measurements.

The proposition has several advantages:

1. The algebraic relationships between operators can be extracted (with some caveats) experimentally from a measurement stream, even though this may be very computationally expensive.

2. Commutators do not require Classical Physics to pre-define the concepts required for use by Quantum Mechanics.

3. Commutators naturally define a "scale" and provide a natural mechanism for the introduction of constants. For example, suppose that

$$[\mathbf{X}_i, \mathbf{X}_j] = g_{ij}(\mathbf{X}_1 .. \mathbf{X}_n)$$

If device $\mathbf{X}_i$ is rescaled ($\mathbf{X}_i \rightarrow f(\mathbf{X}_i)$) then, depending on the algebra, there may be a detectable change in the commutator relations

The commutator relations provide a method of ensuring different separated measuring devices use the same units.

In fact, we propose that meaning can only be assigned to measurements produced by devices that can be calibrated.

10.1.1 Principle of Scale

A set of devices { $\mathbf{X}_i, \ldots \mathbf{X}_n$ } *produces meaningful results only if rescaling is detectable.*

<u>*Example*</u>: Three devices are related by the commutator relationship

$$[\mathbf{L}_x, \mathbf{L}_y] = i\,\hbar\,\mathbf{L}_z$$

$$[\mathbf{L}_y, \mathbf{L}_z] = i\,\hbar\,\mathbf{L}_x$$

$$[\mathbf{L}_z, \mathbf{L}_x] = i\,\hbar\,\mathbf{L}_y$$

Rescale $\mathbf{L}_x \rightarrow \mathbf{L}_x' = \lambda\mathbf{L}_x$, $\lambda \neq 0$, $\lambda \neq 1$ then

$$[\mathbf{L}_x', \mathbf{L}_y] = i\,\lambda\,\hbar\,\mathbf{L}_z$$

$$[\mathbf{L}_y, \mathbf{L}_z] = i\,\hbar\left(\frac{1}{\lambda}\right)\mathbf{L}_x'$$

$$[\mathbf{L}_z, \mathbf{L}_x'] = i\,\lambda\,\hbar\,\mathbf{L}_y$$

The rescaling of $\mathbf{L}_x$ can be partially hidden by the simultaneous rescaling $\mathbf{L}_y$ or $\mathbf{L}_z$. If $\mathbf{L}_z \rightarrow \mathbf{L}_z' = \lambda\,\mathbf{L}_{z.}$, then

$$[\mathbf{L}_x', \mathbf{L}_y] = i\,\hbar\,\mathbf{L}_z'$$

$$[\mathbf{L}_y, \mathbf{L}_z'] = i\,\hbar\,\mathbf{L}_z'$$

$$[\mathbf{L}_z', \mathbf{L}_x'] = i\,\lambda^2\,\hbar\,\mathbf{L}_y$$

However no further scaling $\mathbf{L}_y \rightarrow \mathbf{L}_y' = \mu\, \mathbf{L}_y$, $\mu \neq 0, \mu \neq 1$ (below) can disguise the original scaling.

$$[\mathbf{L}_x', \mathbf{L}_y'] = i\, \mu\, \hbar\, \mathbf{L}_z'$$

$$[\mathbf{L}_y', \mathbf{L}_z'] = i\, \mu\, \hbar\, \mathbf{L}_z'$$

$$[\mathbf{L}_z', \mathbf{L}_x'] = i\, \lambda^2 \left(\frac{1}{\mu}\right) \hbar\, \mathbf{L}_y'$$

Example: Two devices are related by commutator relationship $[\mathbf{x}, \mathbf{p}] = \hbar$.

Rescale $\mathbf{x} \rightarrow \mathbf{x}' = \lambda\mathbf{x}$ (change in the choice of units), then

$$[\mathbf{x}', \mathbf{p}] = [\lambda\mathbf{x}, \mathbf{p}] = \lambda\hbar.$$

The original scaling however can be hidden by rescaling $\mathbf{p}$. I.e. $\mathbf{p} \rightarrow \left(\frac{1}{\lambda}\right)\mathbf{p}$.

Rescale $\mathbf{x}^* = \mathbf{x} + \lambda$ (change in the zero mark of the measuring device), then again

$$[\mathbf{x}^*, \mathbf{p}] = [\mathbf{x}, \mathbf{p}] = \hbar.$$

The choice of a different zero mark has no effect on the commutator relations. I.e. It is possible to replace $\mathbf{x}$ with $\mathbf{x}^*$ with no effect on the physics.

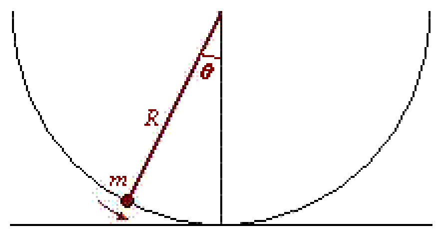
Momentum and position measuring devices cannot be scaled in isolation. One possibility is that momentum and position are not "fundamental quantities" but constructed from other quantities.

11 Time

Recall that naive observers, equipped only with Mach devices, do not have access to a clock. How would such an observer measure time?

11.1.1 Scaling down classical clocks

The diagram below shows a classical pendulum clock with θ as the angle between the pendulum and vertical, the pendulum has length R and mass m.



The system has only a single state variable **θ**, and conjugate momentum ***p***. Time is measured by continually tracking **θ**, and counting the number of times the pendulum crosses the vertical (i.e. the value of $|\theta| = 0$). If a single oscillation is missed the clock becomes inaccurate.

Mechanical clocks do not scale down well into the Quantum world; they all fall prey to Heisenberg's Uncertainty Principle. Any measurement of **θ** would result in uncertainty in ***p***. It would not be possible to continually track the motion of the pendulum; **θ** would then vary randomly, not regularly as required for the device to qualify as a clock.

Perhaps an atomic clock would be a better bet? Atomic clocks use precisely known energy lines to frequency lock microwave radiation; very fast electronics then counts the wave crests of *millions* of photons in a powerful oscillating field. Atomic clocks won't work with a single photon. Atomic clocks do not scale down into the Quantum world any better than mechanical clocks.

11.1.2 Ensemble Clocks

The diagram below shows a quantum clock frequently described in the literature.

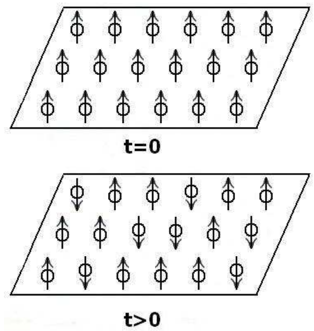


Figure 12
Quantum Clock.

The device consists of a large number of "particles" all prepared in identical states, denoted ↑. The "particles" spontaneously decay to a second state, denoted ↓. Each transition is (suspected to be) statistically independent from any other (electrons will do, photons will not).

A statistical estimate of the time past since the assembly of particles was prepared (t = 0) can be made by counting the number of particles that have changed state (t > 0). A clock can be designed with arbitrarily high confidence by increasing the number of particles in the ensemble.

If $N(t)$ = expected proportion of "particles" in the ↑ state at time t, then

$$N(t) = e^{-\lambda t}$$

Solving for the time t:

$$t \approx - ln(N) / \lambda$$

Ensemble clocks devices provide weak measurements of time. Two identically constructed clocks can produce the different times, although as the number of particles in each clock increases, the discrepancy between the clocks is likely to decrease.

11.1.3 Pauli's Theorem

Pauli's Theorem strongly suggests that there is no such thing as a time operator in Quantum Mechanics. The best attempts at building a quantum clock are statistical in nature. Pauli's Theorem suggests that's as good as it gets.

11.1.4 Problems with a-prior Space-time

Quantum Mechanics is incompatible with the General Theory of Relativity. The theories are fundamentally different in nature. For example, the space-time curvature of General Theory of Relativity is determined by energy density. However in Quantum Mechanics, energy arrives in quanta. Should we expect the space-time curvature to be quantized? Perhaps, but the General Theory of Relativity is a continuous theory. Quantum Mechanics also provides a subjective probabilistic description of nature, but the General Theory of Relativity is an objective deterministic theory.

Yet the Bohr-Einstein Photon-in-a-box thought experiment [3] shows that Quantum Mechanics is consistent with the General Theory of Relativity at least to some degree.

12 The Second Ticker-Tape Conjecture

Time is a statistical concept.

The fact that the "best" quantum clocks produce statistical estimates of "the time" suggests that time may in fact be a statistical concept like temperature or pressure.

13 Hamiltonian

Once we have a clock, we can relax the requirement that transition probability are stable, and that always P(**m** | **m**) = 1. The Hamiltonian can be introduced. See Ariel Caticha[2]

14 Comments

The Copenhagen Interpretation has now become so polluted as a brand that it is now virtually impossible to determine what it is. The Ticker-Tape Interpretation is based on Copenhagen (I would hope that Bohr, Heisenberg and Born would recognise it and approve) but rejects the view that space-time is a necessary pre-requisite for the formulation of physics.

15 String Theory

String Theory is a mathematical generalization of particle Quantum Mechanics. Strings propagate in much the same way as particles do in Feynman's world. If the popular press and String theorists are to be believed, String theory is close to explaining all quantum phenomena. It is the "Theory of Everything".

However String Theory has moved sufficiently far away from standard Quantum theory that the interpretation of String Theory, as opposed to its mathematical content, is less than obvious. What happens when a string is "measured"? Does a position measurement return the "average" position of the string? Or the location of an end-point? If so, which end-point? (The string is a 2D object extended in space - its position cannot be described by a finite set of numbers and therefore cannot be described by a finite set of measurements) What happens if there is no end-point? Does HUP apply to the whole string? Or does it only apply to the "section" of the string being measured? If the "position" of a string is measured with arbitrarily high accuracy, what happens to the momentum of the string? Does the "momentum" become undefined (as opposed to simply unknown)? How does Bell's inequality apply to string theory? Does String theory modify HUP? String theory has moved sufficiently far away from standard Quantum theory that it is hard to know how

David Deutsch, Professor of Physics at the University of Oxford's Centre for Quantum Computing, says "*In the last few decades, many theoretical physicists have assumed that further progress can only come from looking at new mathematical models and then wondering if the models are true representations of nature. An example is String Theory. I think it's unlikely that a research program of that kind can work. Even if you found the right mathematical object, you probably wouldn't even recognise it because you wouldn't know how it corresponds with the real world. For example, if someone had invented quantum theory purely as a mathematical model, how would they ever guess that its multi-valued variables correspond with the quantities that we measure with single values? After all it assigns multiple values to observable quantities simultaneously. I would warn against expecting the answer to come from a new mathematical model. It should be the other way around: first find what you think might be the solution to a problem, then express it as a mathematical model, then test it*".[7]

16 So what have I learnt about Quantum Mechanics?

Much of the content of this document that can be found in various places in textbooks and on the Internet, however it seems to me that, in general, the treatment of the interpretation of Quantum Mechanics by most textbooks is incomplete. Many textbooks, for example, only present enough to contrast Quantum Mechanics with Classical physics or use a semi-historical approach.

There are many reasons why a detailed discussion of interpretation is avoided, but mostly I think the reason is this: Interpretations are controversial. It is impossible to prove an interpretation in the same manner as it is possible to prove the correctness of the predictions of Quantum Mechanics itself. No doubt there is a feeling in many quarters that a discussion on interpretations is ultimately fruitless and would detract from huge areas of practical interest. After all students “pick up” a working interpretation based on the language used to solve problems; few students would feel any need to explore interpretations.

“*Let’s leave interpretations to the philosophers and new-age hippies*”.

Anyway...

Subtle is the Lord. Quantum Mechanics is not “*rife with uncertainty and riddled with paradoxes”* [8], but Quantum Mechanics is so different to everyday experience that the analysis of many situations requires thoughtfulness and care with language.

Only the Copenhagen interpretation is viable. Many-worlds and other interpretations make for rattling good Star Trek stories, but that's not the way the world is.

The London (Ticker Tape) Interpretation of Quantum Mechanics is an extension of the Copenhagen Interpretation. It is deeply rooted in Western Philosophy and hopefully is one of the more “scientific” interpretations in the sense that imposes many constraints on Quantum Mechanics, such as numeric stability, all of which are potentially subject to falsification. It suggests new avenues for research. It will be interesting to see if it is pursued by others.

When I went to University thirty years ago, the interpretation of Quantum Mechanics was regarded as settled, with a few notable holdouts. Has the past three decades produced a deeper understanding of Quantum Mechanics?

There have been some advances. For example, Bell's Inequality is simpler and easier to understand that Von Neumann’s Impossibility Proof, but overall we are in pretty much in the exact same position we were in at the end of the 1930s. That’s not surprising. The physicists of the early 20th Century were not stupid. They almost certainly considered every possible alternative before settling on the Copenhagen Interpretation as the interpretation of choice.

Biologists have discovered that the arguments in favour of Evolution must be continually re-stated to each new generation of biologists; Physicists are discovering that they too must repeat the arguments in favour of Copenhagen.

17 Appendix A - Interpretations

"I think I can safely say no-one understands Quantum Mechanics" – Richard Feynman

Quantum Mechanics is one of the great achievements of mankind. It is at the core of 20^{th} Century technology. Its equations have been verified to unrivalled accuracy. So why do some physicist feel a deep unease?

The answer is that many physicists agree with Richard Feynman. The problem is not that physicists do not understand how to apply the rules of Quantum Mechanics, but rather they do not feel they have a good understanding of why the rules are the way they are. The problem is one of interpretation.

An Interpretation of Quantum Mechanics is an explanation of why Quantum Mechanics is the way it is.

17.1 Why Interpret?

Not everyone believes in the need for interpretation. Feynman famously stated that "no-one understands Quantum Mechanics" but criticised those who felt that the Universe needed to be simple or beautiful. To Feynman, the Universe is the way it is. If a theory is in 100% harmony with observation, then the theory should be accepted as it is. If it is logically consistent, but the rules do not appear to make sense, that's too bad. The universe does not need to appease man.

17.1.1 The Stopping Problem

There is another problem with interpretations: When do we stop? If theory A is the "Theory of Everything" but uses concept B, should we then pursue an explanation for concept B? What if B is something we are not comfortable with?

Alternatively we can turn to Philosophy. Immanuel Kant for example, drew the distinction between the mind and the external world, and between information drawn from the senses and information resulting from the application of logic. Should we expect a "Theory of Everything" to reflect this philosophy? Arguably the Copenhagen Interpretation does; it is based on Positivism. Or is our belief in "common sense" so strong that we should reject any theory that is not based on Realism?

17.1.2 I am not a computer; I am a human being!

"The paradox is only a conflict between reality and your feeling what reality ought to be" – Richard Feynman

If interpretations are so problematic, why do we invest so much time in them? Some reasons include:

- Humans need to find an explanation that they personally feel comfortable with. Manipulating equations is just not satisfying enough. *I.e. there is a psychological need for interpretation.* Psychology however has proven to be a poor guide to the interpretation of Quantum Mechanics.

- It's hard for humans to apply abstract rules - solving problems without intuition is hard. A little intuition goes a long way. *I.e. there is a pragmatic need for interpretation.*

- Quantum Mechanics is unfinished business - there are still unresolved problems. Interpretations hopefully increase our "understanding" of the physics and point the way forward towards new and better theories. Despite Feynman's comments, philosophy and a sense of beauty has had a profound effect on the development of physics, and Feynman's own writing show a deep appreciation of such things.

17.2 Understanding Interpretations

There are essentially 3 types of statements made when discussing interpretations of Quantum Mechanics.

- Predictions of Quantum Mechanics- these cannot be challenged by an interpretation.

- Interpretations, or parts of an interpretation - by definition, these must (a) be consistent with the predictions of Quantum Mechanics, and (b) be an explanation, or part of an explanation, of some part of Quantum Mechanics.

- Speculation - statements which are not any of the above. Any statement that is inconsistent with the predictions of Quantum Mechanics or a restriction on the validity of Quantum Mechanics is speculation.

Statements regarding conditions under which Quantum Mechanics fail are *always* speculation. *There is currently no experimental evidence that standard Quantum Mechanics fails under any conditions, so the issue of conflict between Quantum Theory and observation does not arise.*

A paradox is not an interpretation. It is a situation which, when analysed, shows up some feature of an interpretation or Quantum Mechanics as being incomplete or contradictory or inconsistent with common sense or some branches of Physics. For example, Wigner's Friend posses problems for any of the real waveform interpretations of Quantum Mechanics because the two observers "see" the waveform doing different things.

The Golden Rules

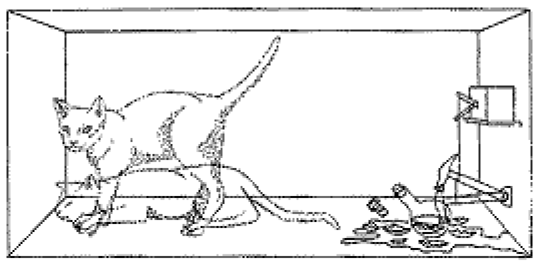
- It is important when analysing statements to determine whether the statement is a prediction of Quantum Mechanics, or part of an interpretation or pure speculation

- When attempting to understand Quantum Mechanics, *speculation is a bad thing.*

Mixing speculation into an argument only results in more speculation. An interpretation mixed with speculation is speculation, not interpretation. Unfortunately authors often skip between prediction, interpretation and speculation, making it very difficult to clearly ascertain the final status of a statement.

17.3 An Example: Schrodinger's Cat

The paradox (a description of a situation): A cat is placed in a box with a radio-active isotope for a short time. An observer agrees to look in the box after some fixed time interval, during which there is a 50:50 chance that the isotope will emit radiation that will be detected by a Geiger counter. If the Geiger counter detects radiation, it will cause a poisonous gas to be released and kill the cat.



Schrodinger's Cat

Prediction of Quantum Mechanics (cannot be challenged by an interpretation): The cat is in a state that is a mixture of alive and dead states until the box is opened and an observer determines the state of the cat.

Copenhagen Interpretation (must be consistent with Quantum Mechanics): The waveform is not real and only reflects our knowledge of the system. The cat could be alive or dead until the observer looks into the box. The paradox presents no problems.

State Vector Interpretation (must be consistent with Quantum Mechanics): The waveform is real, and the cat is therefore really in a state which is "half dead and half alive". But a number of questions arise: What does it mean to be half dead and half alive? Can the cat's status be objectively established? How does opening the box and looking at the cat cause the cat to come back from the half-dead to 100% alive or 100% dead?

The following comments about Schrodinger's Cat are speculation:

- (GRW) The waveform collapses after some short time period for an unspecified reason. Comment: This is not in agreement with standard Quantum Mechanics so is not part of an interpretation; it is speculation.

- The cat is a macroscopic object; the cat is not described by a waveform and the normal rules of Quantum Mechanics do not apply to it. After all, no-one has every seen be able to perform double-slit diffraction experiments using cats. Comment: This is not in agreement with standard Quantum Mechanics so is not part of an interpretation; it is speculation.

- The waveform collapses because common sense dictates that it is so. Comment: This is challenge to Quantum Mechanics so not part of any interpretation and therefore speculation. This comment indicates a complete failure to understand the so-called "scientific method". Common sense does not trump an experimentally established scientific theory.

17.4 Evaluating interpretations

There is no right way to choose the "best" interpretation (psychological factors play an unavoidable part), however the author suggests that the best interpretation (a) are free from contradiction, (b) require the little or no speculation to resolve paradoxes and (c) raises the least unresolved questions (Occam's Razor). In the example above, the Copenhagen Interpretation requires no speculation and does not raise any unresolved questions. The State Vector Interpretation (real waveform) raises a number of questions that cannot easily be resolved. (E.g. what does it mean to be half alive and half dead?). The Copenhagen Interpretation should therefore be objectively preferred to the State Vector Interpretation.

17.5 Ambiguity

The classification of statements into predictions of Quantum Mechanics, interpretations and speculation is extremely helpful in preventing discussions and arguments degenerating into speculative nonsense. The classification however is not as clear cut as it appears; it does not acknowledge that many of the "rules" of Quantum Mechanics are ambiguous.

The Consciousness Causes Collapse interpretation, for example, speculates that the waveform collapses only when it interacts with consciousness. If consciousness causes collapse, then the presence of the cat, which is conscious entity, inside the box causes immediate waveform collapse. However, as stated before, the "standard treatment" of Quantum Mechanics regards the cat as being described as being in a mixed state until the observer looks inside the box. This highlights one of the central problems of Quantum Mechanics: the ambiguous status of waveform and any "collapse".

17.6 Postulates of Quantum Mechanics

Quantum Mechanics can be built from to a small number of postulates using rather sophisticated mathematics. The standard approach is due to John Von Neumann.

The set of axioms used to derive the rest of Quantum Mechanics is related to the choice of physical assumptions; different interpretations are therefore associated with different axiomatic treatments of Quantum Mechanics. However, a full mathematical derivation of Quantum Mechanics would be extremely complex and paradoxically hide much of the physics that we are interested in. It is quite possible to understand the interpretations with resorting to an axiomatic approach, and this is the approach generally pursued in this document.

18 Appendix B – Robertson's Inequality

<u>*Cauchy-Schwartz Inequality*</u>: $<\mathbf{x}, \mathbf{y}> \leq \| \mathbf{x} \| \, \| \mathbf{y} \|$ for all vectors **x**, **y**.

Make the substitution $\mathbf{x} \rightarrow \mathbf{A\Psi}$ and $\mathbf{y} \rightarrow \mathbf{B\Psi}$, then for any vector $\mathbf{\Psi}$, $<\mathbf{A\Psi}|\mathbf{B\Psi}>^2 \leq \|\mathbf{A\Psi}\|^2 \, \|\mathbf{B\Psi}\|^2$

$<\mathbf{A\Psi}|\mathbf{B\Psi}>^2 \; \geq \; | \, im(<\mathbf{A\Psi}|\mathbf{B\Psi}>) \, |^2$

$= | \, \tfrac{1}{2} \, (<\mathbf{A\Psi}|\mathbf{B\Psi}> - <\mathbf{A\Psi}|\mathbf{B\Psi}>^*) \, |$

$= \; \tfrac{1}{2} \, |<\mathbf{A\Psi}|\mathbf{B\Psi}> - <\mathbf{B\Psi}|\mathbf{A\Psi}>|$

$= \tfrac{1}{2} \, |<\mathbf{AB\Psi}|\mathbf{\Psi}> - <\mathbf{BA\Psi}|\mathbf{\Psi}>|$

$= \tfrac{1}{2} \, |<(\mathbf{AB} - \mathbf{BA})\mathbf{\Psi}|\mathbf{\Psi}>|$

$= \tfrac{1}{2} \, E_{\mathbf{\Psi}}([\mathbf{A},\mathbf{B}])$

$\Rightarrow \|\mathbf{A\Psi}\|^2 \, \|\mathbf{B\Psi}\|^2 \; \geq \tfrac{1}{4} E_{\mathbf{\Psi}}([\mathbf{A}, \mathbf{B}])^2$

Make the substitution $\mathbf{A} \rightarrow \mathbf{A} - <\mathbf{A}>$ and $\mathbf{B} \rightarrow \mathbf{B} - <\mathbf{B}>$, then $\|\mathbf{A\Psi}\|^2 = \sigma_{\mathbf{A},\mathbf{\Psi}}$ **and** $\|\mathbf{B\Psi}\|^2 = \sigma_{\mathbf{B},\mathbf{\Psi}}$

$\Rightarrow \sigma_{\mathbf{A},\mathbf{\Psi}} \sigma_{\mathbf{B},\mathbf{\Psi}} \geq \tfrac{1}{2} \, | \, E_{\mathbf{\Psi}}([\mathbf{A}, \mathbf{B}]) \, |$

19 Appendix C – Quantum Mechanics Primer

Quantum Mechanics is one of the 20th century's great achievements. It is however very strange. It is so strange that Richard Feynman famously stated that no one understands it.

At the turn of the 20th Century the world was conceptually a much simpler place. The geometry of Euclid had existed for nearly 2000 years. The physics of Newton, revolutionary in its day, had proved flawless in its predictions for 200 years. Neptune, for example, had been discovered as a result of a search for an unseen planet causing minute discrepancies in the predicted orbit of Uranus. There was no obvious reason to doubt its correctness.

Newtonian physics also was intuitive. Things in the Newtonian world react to forces. People feel forces. They apply force every day of their lives. They are aware when forces are applied to them. They are aware that rigid bodies can transmit forces. They are aware that if a substance is compressed, the material pushes back against whatever is compressing it. They are aware of friction. They are aware, at some level, of abstract principles like Galileo's Principle of Relativity: anyone who has travelled in a car or a boat or an aeroplane knows it's not possible to tell how fast you are going without reference to some external point (e.g. looking out the window).

Only a few, rare individuals had guessed that other options existed and seriously considered them. For example, Georg Riemann climbed the Alps with a theodolite to check that the angles of a triangle did indeed add to 180^o. Fifty years later, Einstein would use Riemann's mathematics to construct the General Theory of Relativity.

19.1 The Atom

The theory of atoms dates back to the ancient Greeks. Leucippus and Democritus (460 BC - 371 BC) proposed that matter is made up of small, eternal, indivisible particles called *atomos*; atoms of the same type being identical in all respects.

John Dalton (1766-1844) and others proposed that atoms could be used to explain the weights and proportions of compounds produced and consumed in chemical reactions. By 1860, the atomic weight of the 60 known elements had been established. In 1869, Mendeleyev and Meyer produced the chemical periodic table, which has since been updated by Ramsay, Mosley and Seaburg.

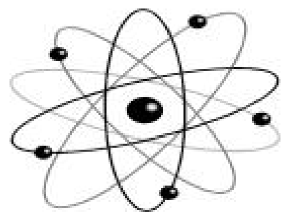
Boyle, Dalton and others used the concept of atoms to explain the properties of gases. Ludwig Boltzmann (1844 - 1906) developed a statistical description of the motion of atoms/molecules in gases.

The existence of atoms was still controversial by the beginning of the 20th Century. Bitter debates, frustration and rejection ultimately led to Boltzmann's suicide (1906). In 1905, Einstein published a paper on Brownian motion. Small grains of pollen can be seen to "judder" about under the microscope as the result of atoms crashing into them at high speed. Within a matter of years of publication, the existence of atoms became universally accepted within the scientific community.

But what do atoms look like?

19.1.1 Version 1 - The classic atom

The picture of the atom taught at school resembles that of the solar system. A heavy nucleus sits at the centre of the atom playing the part of the Sun. Smaller lighter electrons play the part of the planets orbiting the nucleus. The force of gravity is replaced by electrical attraction – each electron carries a single negative charge and is attracted towards the positively charged nucleus.



This "classic" picture of an atom slowly emerged from a series of experiments.

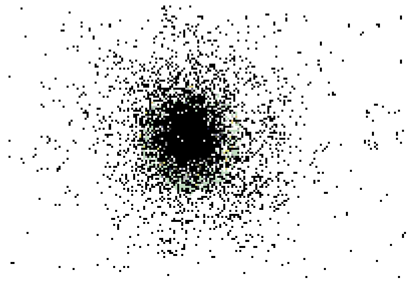
- Thomson demonstrated the existence of negatively charged particles (electrons) using Cathode Ray Tubes.
- Millikan's experiment showed that charge is quantised - charge is lost or gained as electrons are lost or gained.
- Rutherford demonstrated that most of the mass of an atom resided in a small positively charged core by firing alpha particles at thin gold foil. Most of the alpha particles passed through the gold foil as most of the gold atoms consist of empty space. Only a few strike the nucleus, recoiling back at sharp angles.
- Chadwick discovers the neutron (required to explain atomic weights)

Is the classical picture correct?

The classical model is fine for teaching purposes in junior high school. Classical Electromagnetic theory however suggests that the electron should rapidly spiral down and crash onto the nucleus, radiating energy as it goes. That's not what happens - atoms are stable. The model is wrong.

19.1.2 Version 2 - The electron cloud

The second picture of the atom is based on a naive interpretation of Schrodinger's equation from Quantum Mechanics. Atoms are portrayed as a nuclei surrounded by electron clouds. (*Electrons swarm around the nuclei like flies around a cowpat*).



19.1.2.1 Is the electron cloud picture correct?

The electron cloud model is adequate for computational applications, including most of Quantum Chemistry. In particular, it is often possible to use the model to gain an intuitive understanding of atomic energy levels, the spectrum of emitted and absorbed radiation, strength of chemical bonds and material properties.

Unfortunately problems arise again if one looks too closely.

19.2 Quantum Waves

19.2.1 Light is a wave. Or is it?

By the beginning of the 20th century, it was generally regarded that experimental evidence such as diffraction showed light is a wave. Maxwell Equations (1875) predicted waves of Electromagnetic radiation that travel at the measured speed of light. The obvious conclusion was that light is a form of electromagnetic radiation.

But…

Max Plank uses his quantum hypothesis (1901) to explain the spectrum of black body radiation. Einstein produced his light quanta (particle) hypothesis (1905) to explain the Photo-Electric effect.

I.e. There was now experimental proof that light sometimes behave like particles and sometimes behave like waves.

19.2.2 Electrons are particles. Or are they?

J.J. Thompson demonstrated the particle nature of electrons using cathode ray tubes (1897).

But...

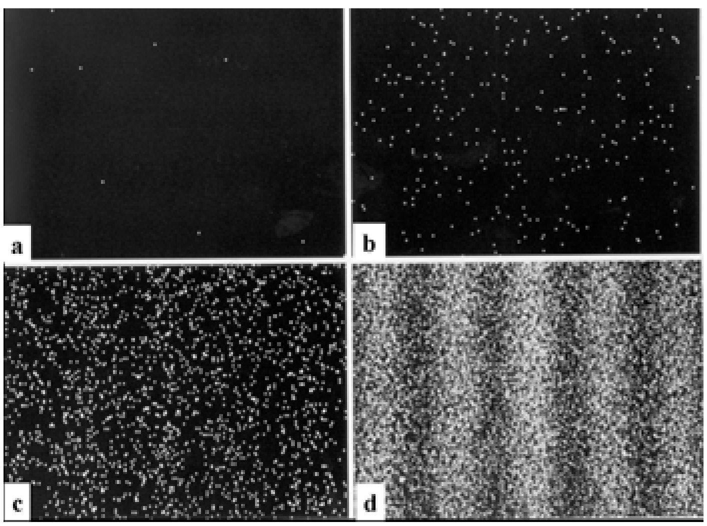
De Broglie (1924) suggests that matter might also behave like a wave in his doctoral thesis, which incidentally would have been rejected had it not been for the intervention of Einstein. Davisson, Germer, Thompson (son of J. J. Thompson) demonstrated electron diffraction in a crystal (1927)

I.e. There was now experimental proof that electrons sometimes behave like particles and sometimes behave like waves.

19.2.3 Schizophrenia

Experimental evidence now indicates that *all* matter can be made to demonstrate both particle and wave like properties. How can the two behaviours be reconciled?

The motion of a particle is described by a wave moving through space. It can be diffracted. Whenever energy is transferred from the wave, the energy is transferred as a quanta. Quantum Mechanics describes the motion of the wave. The transfer of energy from the wave cannot be predicted precisely, the best that Quantum Mechanics can do is to calculate probabilities of an interaction.



The diagram at left shows the result of a double slit diffraction experiment. Quantum Mechanics predicts a waveform with alternating bands of high and low intensity. Frames (a)-(d) show snapshots over time as individual particle "arrive" (transfer energy) to the screen. Initially it is difficult to see any pattern (a)-(b). As time goes on, the waveform reveals itself (d).

It is tempting to think of electrons "hiding" inside the wave, only emerging when interacting with some other part of the world, but even that picture turns out to be too simple.

19.2.4 Formalism

Quantum waveforms are complex valued and interfere destructively. Probability waves are always "positive" and cannot interact destructively. If $\mathbf{\Psi(x,t)}$ is the waveform describing the state of a particle, then the position probability distribution $\mathbf{P(x,t)}$ is given by the formula

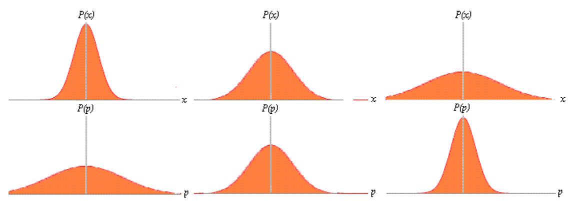
$$\mathbf{P(x,t)} = |\, \mathbf{\Psi(x,t)} \,|^2$$

Why? *A very good question*. If you can come up with a really good reason for this, you will have solved one of the truly great mysteries of the Universe.

Dirac's bra-ket notation is sometimes used to describe waveforms. For example, the waveform describing an electron might be denoted **|e**$^-$ >, the waveform describing a sperm whale might be denoted **|whale>.**

19.3 Heisenberg's Uncertainty Principle (HUP)

Heisenberg's Uncertainty Principle lies at the heart of Quantum Mechanics.



To see that this is so, ask the question: Why are we forced to only deal with probabilities? In the case of an electron moving through space, why can we not build a measuring device and just measure the position and momentum of the electron? Once both the position and momentum are known, then it possible in theory (at least in classical theory) to calculate the position of the particle precisely into the future.

There would not seem to be any obvious reason why this could not be done, although the engineering problems may be considerable.

Heisenberg's Uncertainty Principle says otherwise: ***It is not possible to obtain exact simultaneous measurements of a particle's position and momentum***.

More precisely, Heisenberg's Uncertainty Principle looks like

$$\sigma_p.\sigma_x \geq \frac{\hbar}{2}$$

where σ_x is the standard deviation of the probability distribution of a particle's position (**x**), σ_x is the standard deviation of the probability distribution of the particle's momentum (**p**), h is Plank's constant = 6.625 x 10^{-34} J-s. HUP guarantees that the more "precise" the information available about a particle's position is, the greater the uncertainty in the particle's momentum. Even if the particle's momentum had previously been determined with a high degree of accuracy, any measurement of the position would destroy the validity of the first measurement of momentum. Similarly the more precise the information about a particle's momentum is, the greater the degree of uncertainty in the particle's position. It is simply not possible to make accurate measurements of both position and momentum at the same time.

Heisenberg's Uncertainty Principle applies to all matter including sub-atomic particles, people, cars, galaxies, sperm whales and bowls of Petunias. Plank's constant is very small. At normal energy (and momentum) levels, the uncertainty in position of an electron is (not surprisingly) roughly the size of an atom.

19.3.1 Absorption and Emission of Radiation

Quantum Mechanics predicts that the energy levels of an atom are quantised; the energy levels are characterised by quantum numbers.

Chemists use the misleading term "orbitals" to describe the waveforms associated with each energy level. They also, for historical reasons, refer to the orbitals by letter (s, p, d, f …) rather than quantum number. The figure below shows the shape of the various orbitals. Note however that the orbitals do not have sharp boundary; rather the probability of finding an electron gradually drops off with distance from the nucleus.

TODO
figure 14.3.1

Protons and neutrons in atomic nuclei behave in similar way to electrons in an atom; models using orbitals can be used to predict the stability of atomic nuclei. The pattern of stability is not the same as that of the periodic table since the energy levels associated with the nuclear orbitals do not match their atomic equivalents, and therefore each nuclear shell contains a different number of orbitals than the equivalent atomic shell.

Atoms randomly make transitions from high energy states to low energy states. If an atom starts in a state with energy E_0 and ends up in a state with energy E_1, then the difference in energy E_0 - E_1 will be carried away as a photon. The result is that radiation (radio waves, micro-waves, infra-red, light, ultra-violet, x-rays) emitted by an atom have very precise frequencies, giving rise to characteristic spectra.

The process works in reverse. Atoms may randomly absorb passing photons, but only if the energy of the photon matches the difference between its existing energy state and a higher energy state. If the photon is absorbed, the atom goes into the higher energy state.

19.3.2 Tunnelling

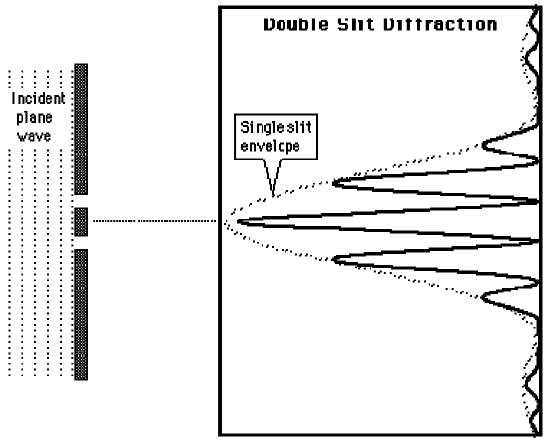
In general, the probability distribution of finding a particle at *any* point in space is not zero; rather it tapers off slowly approaching zero as the distance from the distribution mean increases. That allows particles to tunnel through any barrier, even the event horizon of a black hole. However the probability of a macroscopic object like a car tunnelling through a barrier like a brick wall is almost zero.

A version of Heisenberg's Uncertainty Principle applies to measurements of time and energy.* Mass is energy. A vacuum can create particles out of nothing - provided that they do not live longer than that the time interval predicted by Heisenberg.

* Strictly speaking the Energy-Time uncertainty relationship is only analogous to Heisenberg.

19.3.3 Double slit Diffraction

Double slit diffraction is the example of quantum behaviour par excellence. Feynman stated that if it was possible to understand diffraction, then it was possible to understand all of Quantum Mechanics. A double slit diffraction apparatus consists of (1) a particle source - typically a light or electron source, (2) a screen with 2 parallel slits through which particles can pass, and (3) a screen where the particles that have passed through the double slits arrive. The characteristic double slit pattern is shown in the diagram below and can be understood as a result of waves interfering with each other.

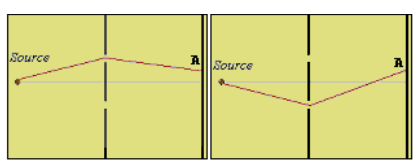


Double Slit Diffraction

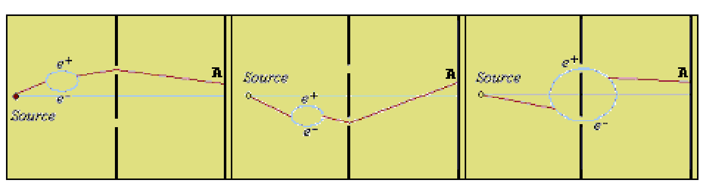
19.3.3.1 Which slit did the electron go through?

It is tempting to consider the possibility that the interference pattern results from particles going through one slit interfering with the motion of particles going through the other slit, however it is possible to set the apparatus up so that statistically only one particle passes through the double slit apparatus at a time; the standard double slit interference pattern is still seen. That means that the photon or electron interferes with itself!

Richard Feynman came up with an intuitive method of calculating the probabilities of any event (E.g. the particle in a double slit diffraction arriving at the point A in the diagram below): (1) Draw diagrams showing every possible interaction, (2) calculate the amplitude of the event given the interactions in the diagram, and (3) add the amplitudes for each diagram together. The existence of virtual particles significantly increases the complexity of calculations.



1st order Feynman Diagrams



Some 2nd order Feynman Diagrams (1 virtual particle interaction)

19.4 Special Relativity

Special Relativity is based on the tenet that the speed of light in a vacuum is always constant (designated by the letter c). This simple idea has some startling consequences. The main results relevant to Quantum Mechanics are

- Time passes at different rates for observers travelling at different velocities.
- Events which appear simultaneous in one frame of reference will not be simultaneous in a frame of reference moving relative to the first.
- Minkowski diagrams (space-time diagrams) can be used to represent the structure of space-time.

19.5 Why is the speed of light in a vacuum constant?

Mach Principle – No preferred frame of reference. Predicted by Maxwell's Equations. Only Einstein accepted the equations at face value.

19.6 The measurement of time

The diagram below shows light clocks. The clocks measure the length of time it takes for a pulse of light to travel from the top surface to the bottom surface and back again.

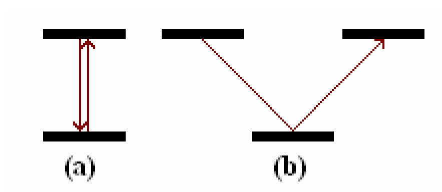


Figure (a) shows a "stationary" light clock
Figure (b) shows a light clock moving to the right

The clock on the left is stationary, while the clock on the right is identical in manufacture except that is moving to the right with a velocity *v*. Simple algebra reveals that the moving clock runs slow (the light has further to travel) by a factor

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Similar diagrams are shown below for clocks that move in the direction of the internal light beam. Simple algebra reveals the "moving" clock on the right also runs slower, but at a different rate. The difference can be put down to the foreshortening of moving objects.

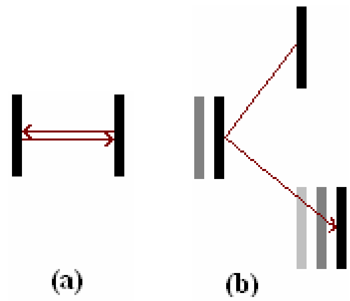


Figure (a) shows a "stationary" light clock
Figure (b) shows a light clock moving downwards

19.7 Lorentz Transformations

The result is that observers in different frames of reference see moving objects foreshortened, and time passes more slowly for moving observers. The mathematical relationship between "equivalent" coordinate system used by observers is

$$x' = x\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = t\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

19.8 Minkowski Diagrams

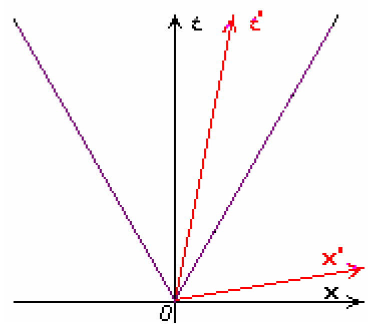


Diagram showing different coordinate grid imposed on space-time.

19.8.1 Simultaneity - The Train in a Tunnel Paradox

19.9 Feynman's Rule

Quantum Mechanics does not change the rules of probability for observable events. However the following rules used to calculate of probabilities of "virtual" events is very different.

<u>*Classical probability theory:*</u> If **A** and **B** are mutually exclusive events, then

$$P(\mathbf{A} \text{ or } \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B})$$

<u>*Feynman's probability rules*</u> : (taken from 1-7 of The Feynman Lectures on Physics) "*The probability of an event in an ideal experiment is given by the square of the absolute value of a complex number Ψ_A, which is called the probability amplitude*"

$$P(\mathbf{A}) = |\Psi_{\mathbf{A}}|^2$$

"*When an event can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes of each way separately. There is interference.*"

$$P(\mathbf{A} \text{ or } \mathbf{B}) = |\Psi_{\mathbf{A}} + \Psi_{\mathbf{B}}|^2$$

"*If an experiment is performed which is capable of determining whether one or another alternative is actually taken, the probability of the event is the sum of probabilities of each alternative. The interference is lost*"

$$P(\mathbf{A} \text{ or } \mathbf{B}) = |\Psi_{\mathbf{A}}|^2 + |\Psi_{\mathbf{B}}|^2$$

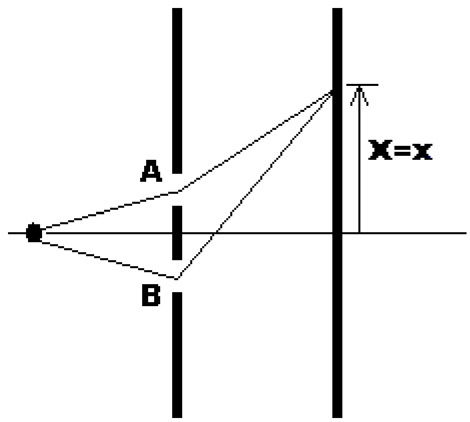
A similar rule applies to the conjunctions of *independent* propositions:

$$P(\mathbf{A} \text{ and } \mathbf{B}) = |\Psi_{\mathbf{A}} . \Psi_{\mathbf{B}}|^2$$

Example: The diagram below shows a classic double-slit diffraction experiment. Let **A** be the proposition "The particle goes to **x** through slit A" and **B** be the proposition "The particle goes to **x** through slit B". The various possible alternative ways an event can happen are sometimes called "histories". I.e. **A** and **B** are alternative histories.

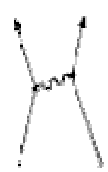
Feynman's rule:

$$P(\text{particle arriving at } \mathbf{x}) = P(\mathbf{A} \text{ or } \mathbf{B}) = |\Psi_\mathbf{A} + \Psi_\mathbf{B}|^2$$



Double Slit Diffraction

Feynman diagrams are used to analyse particle interactions. The initial and final states are determined, and diagrams are "drawn" for each possible interaction leading to the desired outcome. During these interactions, particles can do whatever is possible, and do. Photons, for example, routinely travel faster or slower than the speed of light. Virtual particles are created and destroyed. Each unique topology + choice of coordinates for events (i.e. graph vertex) yields a possible history. Summing the amplitudes over all possible histories yields the probability of the original interaction.



Feynman Diagram

Feynman's rules are fundamental to modern theories such as QED and QCD, however there are technical problems. The sum of Feynman histories for a given particle interaction may not converge to a finite result. Results can still be obtained by use of "Renormalisation" however the process is mathematically dodgy. (Basically just stopping counting at some level of diagram complexity and "adjust" the numbers so the known values match with experiment) Some physicists attribute this failure to the process of summing diagrams down beyond the Planck length (10^{-33} cm). Despite problems, the Feynman approach is stunningly successful. E.g. the QED calculation of magnetic moment of an electron agrees with experiment to within one part in 10^8.

20 Angular Momentum

20.1 Commutator Relations

Quantum Mechanical operators $\mathbf{L}_x$, $\mathbf{L}_y$, $\mathbf{L}_z$ measure angular momentum if they satisfy the commutator relations

$$[\mathbf{L}_x, \mathbf{L}_y] = i\hbar\, \mathbf{L}_z,$$

$$[\mathbf{L}_y, \mathbf{L}_z] = i\hbar\, \mathbf{L}_x,$$

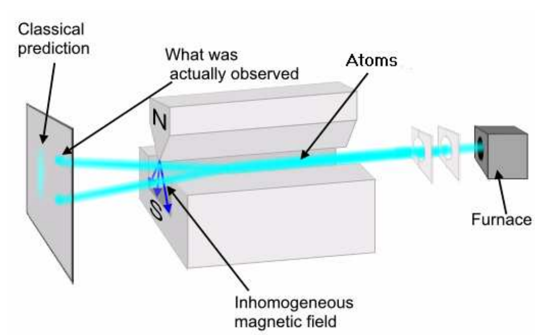
$$[\mathbf{L}_z, \mathbf{L}_x] = i\hbar\, \mathbf{L}_y$$

21 Intrinsic Spin

Observation has shown that particles have "intrinsic" spin (units of angular momentum).

21.1 Spin ½ Particles

Experiments (e.g. Stern-Gerlach) have shown that electrons, some atoms and other sub-atomic particles have an intrinsic "2-valued" angular momentum.



21.1.1 Pauli Spin Matrices

There are 3 distinct orientations in space (denoted x, y and z), and there are 2 distinct values (denoted "up" and "down")

Pauli Matrices provide a model for the measurement of angular momentum in each of the 3 standard spatial orientations. Standard representation and resulting eigenvectors are shown below.

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \left|Up(x)\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \lambda = 1 \qquad \left|Down(x)\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \lambda = -1$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \left|Up(y)\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}, \lambda = 1 \qquad \left|Down(y)\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}, \lambda = -1$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \left|Up(z)\right\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \lambda = 1 \qquad \left|Down(z)\right\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \lambda = -1$$

21.1.1.1 Algebraic properties

$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$

$\sigma_x . \sigma_y = i.\sigma_z$

$\sigma_y . \sigma_z = i.\sigma_x$

$\sigma_z . \sigma_x = i.\sigma_y$

$[\sigma_x, \sigma_y] = 2i.\sigma_z$

$[\sigma_y, \sigma_z] = 2i.\sigma_x$

$[\sigma_z, \sigma_x] = 2i.\sigma_y$

21.1.2 Relationship between Pauli matrices and spin ½

Put $\mathbf{L}_x = \lambda.\sigma_x$, $\mathbf{L}_y = \lambda.\sigma_y$, $\mathbf{L}_z = \lambda.\sigma_z$, then

$$[\mathbf{L}_x, \mathbf{L}_y] = [\lambda.\sigma_x, \lambda.\sigma_y] = |\lambda|^2 [\sigma_x, \sigma_x] = |\lambda|^2.2.i.\sigma_z$$

$$\text{But } [\mathbf{L}_x, \mathbf{L}_y] = i\hbar\, \mathbf{L}_z = i\hbar\, (\lambda.\sigma_z)$$

$$\Rightarrow \lambda = \hbar/2$$

$$\mathbf{L}_x = \hbar/2.\sigma_x, \mathbf{L}_y = \hbar/2.\sigma_y, \mathbf{L}_z = \hbar/2.\sigma_z$$

σ_x, σ_y and σ_z have known eigenvectors, with eigenvectors $\lambda = \pm 1$.

$\mathbf{L}_x$, $\mathbf{L}_y$ and $\mathbf{L}_x$ have the same eigenvectors, with eigenvectors $\lambda = \pm$ ½.ħ.

I.e. the particle spin of ± ½.ħ is the result of the commutator relations.

21.1.3 Quarterions

These are closely related to Pauli matrices and sometime appear in the literature in their place. Quarternions are algebraic extensions to Complex numbers.

$$i^2 = j^2 = k^2 = -1.$$

$$i.j.k = 1$$

$$i.j = k$$

$$j.k = i$$

$$k.i = j$$

21.1.3.1 Quarternions - Preferred representation

$$\mathbf{i} = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} = i.\sigma_x \qquad \mathbf{j} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = i.\sigma_y \qquad \mathbf{k} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = i.\sigma_z$$

21.2 Spin 1 Particles

Spin 1 particles have "3-valued" angular momentum. Standard representations are shown below for the angular momentum operator and eigenvectors are shown below.

$$\mathbf{L}_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\left|L_x = 1\right\rangle = \frac{1}{2}\begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}, \lambda = \sqrt{2}$$

$$\left|L_x = 0\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \lambda = 0$$

$$\left|L_x = -1\right\rangle = \frac{1}{2}\begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}, \lambda = -\sqrt{2}$$

$$\mathbf{L}_y = \frac{\hbar}{2}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

$$\left|L_y = -1\right\rangle = \frac{1}{2}\begin{bmatrix} -i \\ \sqrt{2} \\ i \end{bmatrix}, \lambda = \sqrt{2}$$

$$\left|L_y = 0\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \lambda = 0$$

$$\left|L_y = -1\right\rangle = \frac{1}{2}\begin{bmatrix} i \\ -\sqrt{2} \\ -i \end{bmatrix}, \lambda = -\sqrt{2}$$

$$\mathbf{L}_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\left|L_z = 1\right\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \lambda = 1$$

$$\left|L_z = 0\right\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \lambda = 0$$

$$\left|L_z = -1\right\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \lambda = -1$$

21.3 Field Angular Momentum (Orbital Angular Momentum)

If a system is described by state $\Psi(x, y, z)$, then.

$$\mathbf{L}_x = i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$\mathbf{L}_y = i\hbar(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z})$$

$$\mathbf{L}_z = i\hbar(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x})$$

21.4 Algebraic Properties of Angular Momentum

Define total angular momentum operator $\mathbf{L}^2 = \mathbf{L}_x^2 + \mathbf{L}_y^2 + \mathbf{L}_z^2$, then

$$[\mathbf{L}^2, \mathbf{L}_x] = [\mathbf{L}^2, \mathbf{L}_y] = [\mathbf{L}^2, \mathbf{L}_z]$$

$\mathbf{L}^2$ and $\mathbf{L}_z$ commute so $\mathbf{L}^2$ and $\mathbf{L}_z$ are simultaneously measurable. Define $| jm >$ as the state where

$$\mathbf{L}^2 | jm > = j | jm > \text{ and } \mathbf{L}_z | jm > = m | jm >$$

Define "ladder operators" $\mathbf{L}_+ = \mathbf{L}_x + i.\mathbf{L}_y$ and $\mathbf{L}_- = \mathbf{L}_x - i.\mathbf{L}_y$, then

$$[\mathbf{L}^2, \mathbf{L}_+] = [\mathbf{L}^2, \mathbf{L}_-] = 0$$

$\mathbf{L}_+$ and $\mathbf{L}_-$ commute with $\mathbf{L}^2$ so are simultaneously measureable.

$$\mathbf{L}_+ | jm > = | jm_+ > \text{ and } \mathbf{L}_- | jm > = | jm_- > \text{ for some } m_+, m_-$$

How does a measurement of $L_\pm$ affect the z-momentum?

$$\mathbf{L}_z \mathbf{L}_\pm | jm > = [\mathbf{L}_z, \mathbf{L}_\pm] | jm > + \mathbf{L}_\pm \mathbf{L}_z | jm >$$

$$= (m \pm \hbar) | jm >$$

Note $-j < m < j$. The choice of z direction is arbitrary. If m is a possible value for intrinsic angular momentum in some direction, then so is (m + ħ) provided $-j < m + \hbar < j$.

Similarly $(m - \hbar)$ is a possible value provided $-j < m - \hbar < j$.

In the case of quantised angular momentum, possible values are shown in the pyramid below.

0

-½ ħ ½ ħ

-ħ 0 ħ

-3/2 ħ ½ ħ ½ ħ -3/2 ħ

-2ħ -ħ 0 ħ 2ħ

22 Into the 4th Dimension

22.1 The Dirac Equation

Relativity implies that for a single particle:

$$\boldsymbol{p}^2 - m^2c^2 = 0$$

where $\boldsymbol{p}$ is the 4-momentum. The Quantum Mechanical version of this is (Klein-Gordon)

$$(\partial_\mu{}^2 - m^2c^2)\Psi = 0$$

Assume that the particle waveform can be factored into separate 1st order equations. I.e.

$$(\gamma_\mu.\partial_\mu - imc)\,(\gamma_\mu.\partial_\mu + imc)\Psi = 0$$

where γ_μ. (μ=0,1,2,3) are constants.

$$(\gamma_\mu.\partial_\mu - imc)(\gamma_\tau.\partial_\tau + imc)\Psi = 0$$

$$\Rightarrow (\gamma_\mu.\gamma_\tau.\partial_\mu\partial_\tau - m^2c^2)\Psi = 0$$

This reduces to the Klein-Gordon equation if

$$\gamma_\mu\,\gamma_\tau + \gamma_\tau\,\gamma_\mu = 2\delta_{\mu\tau} \qquad \text{for all } \mu,\tau=0,1,2,3$$

γ_τ cannot be Real or Complex numbers.

Possible representations of γ_k, γ (4x4 matrices):

$$\gamma_0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \qquad \gamma_k = \begin{bmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{bmatrix} \text{ for } k = 1,2,3$$

22.2 4-Dimensional Pauli Matrices

The Pauli spin matrices are a canonical representation of a system with three conjugate 2-valued properties.

What would a system with 4 conjugate 2-valued property, denoted by **K**, look like?

We can use the existing Pauli representation for the first three 2-valued properties and look for a representation for the fourth.

Let $\mathbf{\Psi} = \begin{bmatrix} a \\ b \end{bmatrix}$ be an eigenvector of K, then P($\mathbf{\Psi}$ | $\mathbf{e}_i$) for all 6 Pauli eigenvectors.

$$\begin{bmatrix} a \\ b \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = a = \frac{1}{\sqrt{2}} \Rightarrow |a| = \frac{1}{\sqrt{2}} \quad \text{... (1)}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} = b = \frac{1}{\sqrt{2}} \Rightarrow |a| = \frac{1}{\sqrt{2}} \quad \text{... (2)}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(a+b) = \frac{1}{\sqrt{2}} \Rightarrow |a+b| = 1 \quad \text{... (3)}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} . \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(a-b) = \frac{1}{\sqrt{2}} \Rightarrow |a-b| = 1 \text{... (4)}$$

$\mathbf{\Psi} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm j \end{bmatrix}$ would be a solution for equations (1) ... (4) where *j* is a unit quarternion.

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm j \end{bmatrix} . \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm i \end{bmatrix} = \frac{1}{2}(1 + i.j) = \frac{1}{\sqrt{2}} \Rightarrow |1 + i.j| = |1 + k| = \sqrt{2} \quad \text{... (5),(6)}$$

Note that *i, j, k* must be introduced together since if $i.j \neq i$ then

$$i.j - i = 0 \Rightarrow i.(j-i) = 0 \Rightarrow j = i$$

For symmetry purposes, choose eigenvectors

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm j \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm k \end{bmatrix}$$

What would the operators look like? Denote the 4 dimensional operators $\mathbf{K}_0$, $\mathbf{K}_1$, $\mathbf{K}_2$, $\mathbf{K}_3$.

We know the standard representation for **K**1 (standard Pauli matrix) is $\mathbf{K}_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$. By symmetry, the complete representation is

$$\mathbf{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{K}_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}, \mathbf{K}_3 = \begin{bmatrix} 0 & -k \\ k & 0 \end{bmatrix}$$

Recall the standard representation

$$\boldsymbol{i} = i.\sigma_x, \boldsymbol{j} = i.\sigma_y, \boldsymbol{k} = i.\sigma_z$$

This implies

$$\mathbf{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{K}_1 = \begin{bmatrix} 0 & -i.\sigma_x \\ i.\sigma_x & 0 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0 & -i.\sigma_y \\ i.\sigma_y & 0 \end{bmatrix}, \mathbf{K}_3 = \begin{bmatrix} 0 & -i.\sigma_z \\ i.\sigma_z & 0 \end{bmatrix}$$

I.e.

$$\mathbf{K}_0 = \boldsymbol{\gamma}_0,$$

$$\mathbf{K}_1 = -i.\boldsymbol{\gamma}_1,$$

$$\mathbf{K}_2 = -i.\boldsymbol{\gamma}_2,$$

$$\mathbf{K}_3 = -i.\boldsymbol{\gamma}_3$$

where $\boldsymbol{\gamma}_\alpha$ are the Dirac matrices.

A system of 4 conjugate 2-valued properties is naturally described by 4 vectors and 4 dimensional Dirac matrices! And Relativity + Dirac matrices leads to anti-matter!

23 Appendix E – M5 Software

23.1 Software Documentation

C# Software can be downloaded from http://www.quantum.bowmain.com which demonstrates the Principle of Relativity and the extraction of Commutator relationships from Mach devices using simulations. Some expertise in C# to understand is required to understand the code.

Visual C# Express Version can be downloaded free of charge from Microsoft.

The software contains 3 modules:

- Mach.dll
- Experiment1.exe
- Experiment2.exe

The Mach framework contains the following objects and interfaces:

- **MeasurmentDevice** – abstract class. Derived classes must implement **MeasureInternal**() which simulates a measurement. The returned value is rounded by an amount specified by **MeasurementDevice**.**Resolution**.

- **Measurement** – consists of a **MeasurementDevice** and a measurement value.

- **History** – a list of **Measurement**.

- **QuantumState** – a list of **Measurement**. A **QuantumState** is typically a sub-list of some **History**.

- **ISystemState** interface - This interface is implemented by objects that simulate systems "under investigation" by measurement devices. It contains the single method **OnNewTime(double t)** which allows the system to evolve over time.

- **INotifyMeasurement** interface – This interface is implemented by objects that simulate "observers". It is used by the program to gather statistics.

- **Universe** – contains separate lists of **ISystemState**, **INotifyMeasurement**, **MeasurementDevice**. Run the simulation – randomly increments the time, selects a **MeasurementDevice** and make a measurement. Notifies the observers.

- **Matrix, Vector, Statistics, QuasiRandomNumberGenerator** – support objects.

Experiment1.exe demonstrates the extraction of Commutator relations for **HeisenbergDeviceA** and **HeisenbergDeviceB** (both derived from **MeasurementDevice**)

Experiement2.exe demonstrates the Principle of Relativity (**M5**). The Commutator relations for **ClassicalDevice0** and **ClassicalDevice1** (both derived from **MeasurementDevice**) are zero. The zip file also contains Experiment2.xls which shows the results of the simulation for 1000, 5000 and 10000 measurements.

24 References

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11. “Consistency and Linearity in Quantum Theory”, Ariel Caticha, 1998.

25 Thanks

1. Tx to Robert Watson, Peter Kirby

26 Feedback

If you find an inaccuracy in any of the above please e-mail me at sokane@bowmain.com. I am quite happy to expand the above to include alternative interpretations also.